Atomic Energy Control Board
P.O. Box 1046
Ottawa, Canada
K1P 5S9

Commission de contrôle de l'énergie atomique
C.P. 1046
Ottawa, Canada
K1P 5S9

INFO-0190

MODE SHAPE AND NATURAL FREQUENCY IDENTIFICATION FOR SEISMIC ANALYSIS FROM BACKGROUND VIBRATION

by

S. Bhan and Z. Wozniak
Acres International Limited
Niagara Falls, Ontario

A research report prepared for the
Atomic Energy Control Board
Ottawa, Canada

February 1986

Canada

Research report

86-05424/

ABSTRACT

The feasibility of calculating natural frequencies and mode shapes of major equipment in a CANDU reactor from the measurements of their response to background excitation has been studied. A review of vibration data measured at various locations in CANDU plants shows that structures responded to a combination of random and harmonic background excitation. Amplitude of measured vibration is sufficient to allow meaningful data analysis. Frequency content in the 0 to 50-Hz range, which is of interest for earthquake response, is present in some of the vibration measurements studied.

Spectral techniques have been developed for determining the response function of structures from measured vibration response to background excitation. The natural frequencies and mode shapes are then evaluated graphically from the frequency function plots. The methodology has been tested on a simple cantilever beam with known natural frequencies and mode shapes. The comparison between the theoretical and the computed natural frequencies and mode shapes is good for the lower modes. However, better curve-fitting techniques will be required in future, especially for higher modes.

Readily available equipment necessary for the measurement of background vibration in a CANDU plant (which is commercially available) has been identified. An experimental program has been proposed to verify the methodology developed in this study. Recommendations are also made to study methods to improve the accuracy of the mode shape and natural frequency prediction.

RÉSUMÉ

On a étudié la possibilité de calculer la fréquence naturelle et la configuration du mode de vibration pour l'équipement principal d'un réacteur nucléaire CANDU, se basant sur la mesure de la réponse à l'excitation de fond. Un examen des données sur la vibration de l'équipement mesurée à plusieurs endroits dans les centrales nucléaires CANDU fait voir que les structures répondent à une combinaison d'excitation de fond tout-venant et harmonique. L'amplitude de la vibration mesurée est suffisante pour permettre une analyse valable des données. Dans quelques-unes des mesures de vibration qu'on a examinées, on a trouvé un taux de fréquence dans la gamme de 0 à 50 Hz, ce qui est important pour la réponse aux tremblements de terre.

On a mis en valeur des techniques spectrales afin de déterminer la fonction de réponse d'une structure sur la base de mesures de la vibration qui se produit comme réponse à une excitation de fond. Ensuite, on a determiné par représentation graphique les fréquences naturelles et les configurations du mode à partir des relevés de la fonction de réponse. On a fait l'essai de cette méthode avec une poutre en porte-à-faux simple, dont on savait les fréquences naturelles et les configurations du mode. On a constaté que, dans le cas de modes moins élevés, la comparaison entre les fréquences naturelles et les configurations du mode théoriques et les valeurs qu'on a calculées est assez favorable. Cependant, pour les modes plus élevés, il sera nécessaire de trouver des meilleures techniques pour l'ajustement des courbes.

On a identifié l'appareillage nécessaire qui est disponible sur le marché pour mesurer les vibrations de fond dans une centrale CANDU. On a proposé un programme expérimental ayant pour but la vérification de la méthode mise au point dans la présente étude. De plus, on a apporté des recommandations pour l'étude des méthodes qui amélioreraient la précision des prédictions de la configuration du mode et de la fréquence naturelle.

DISCLAIMER

The Atomic Energy Control Board (AECB) is not responsible for the accuracy of the statements made or opinions expressed in this publication and neither the Board nor the authors assume liability with respect to any damage or loss incurred as a result of the use made of the information contained in this publication.

This work was produced by Acres International Limited (Acres) under contract to the AECB, Contract No. R1103424344.

NOMENCLATURE

c	Damping coefficient
f	Frequency
h	Impulse response function
j	$\sqrt{-1}$
k	Stiffness
m	Mass
t	Time
$tr(S)$	Trace of the matrix S
u_{ij}	Elements of eigenvector
x	Displacement
$\dot{x}$	Velocity
$\ddot{x}$	Acceleration
y	State variable
A	Constant
B	Frequency bandwidth
C_i	Modal damping
F_i	Force
H_{ij}	Frequency response function
Im	Imaginary part
K_i	Modal stiffness
M_i	Modal mass
P_i	Modal force
R_{ii}	Autocorrelation function
R_{ij}	Cross-correlation function
Re	Real part

S_{ii}	Spectral density function
S_{ij}	Cross-spectral density function
Z_i	Principal variable
α	Constant
β	Constant
ϵ	Error
γ	Coherence function, structural (hysteretic) damping factor
Ψ	Phase
ξ	Dimensionless damping ratio
λ_i	Eigenvalue
w	Angular frequency
w_i	Natural frequency
τ	Lag time
Δ	Difference

MATRIX NOTATION

$[C]$	Damping matrix
$[\ C\ ]$	Modal damping matrix
$\{F\}$	Force vector
$[G]$	Impulse response function matrix
$[H]$	Frequency response function
$[K]$	Stiffness matrix
$[\ K\ ]$	Modal stiffness matrix
$[M]$	Mass matrix
$[\ M\ ]$	Modal mass matrix
$\{P\}$	Modal force
$[S]$	Spectral density function matrix
$[\phi]$	Modal matrix

$\{u\}$	Eigenvector
$\{x\}$	Displacement vector
$\{y\}$	State variable vector

TABLE OF CONTENTS

ABSTRACT..... i

RESUME..... i

DISCLAIMER ii

NOMENCLATURE..... iii

A. INTRODUCTION..... 1

B. LITERATURE REVIEW..... 2

C. PLANT VIBRATION MEASUREMENTS..... 3

1. General..... 3

2. Liquid Zone Control Unit..... 3

3. Feeder Cabinet..... 4

4. Feeder Pipe..... 4

5. Feedwater Heater..... 4

D. METHOD OF ANALYSIS..... 5

E. EVALUATION OF MODEL CHARACTERISTICS OF A CANTILEVER BEAM..... 10

F. EQUIPMENT REQUIREMENTS..... 26

G. EXPERIMENT DESIGN..... 26

H. CONCLUSIONS..... 30

I. RECOMMENDATIONS..... 31

J. ACKNOWLEDGEMENT..... 31

REFERENCES

BIBLIOGRAPHY

APPENDICES:

TYPICAL PLANT VIBRATION DATA.....APPENDIX A

BACKGROUND THEORY.....APPENDIX B

COMPUTER PROGRAMS.....APPENDIX C

MODE SHAPE AND NATURAL FREQUENCY IDENTIFICATION FOR SEISMIC ANALYSIS FROM BACKGROUND VIBRATION

A. INTRODUCTION

The design of structures and equipment such as the steam generators, calandria and fueling machine in a CANDU plant includes verification that all components can safely withstand a design earthquake. Large mathematical models are formulated to analytically predict the dynamic characteristics of the equipment which are then used to determine response to the design earthquake. The cost of performing this analysis is very high. The accuracy of the results is often questionable because of the complexity of the structures and the boundary conditions. Many pivotal parameters such as the representation of soil-structure interaction, damping and mechanical joint characteristics are unknown at the design stage although their effect on structure dynamics performances is significant.

A field verification of the analytical models is therefore desirable for a satisfactory seismic qualification. Usually, dynamic testing of large structures requires large and expensive shakers which artificially excite the structure at known vibration frequencies.[Ref. 1] The structural response acceleration and the exciting force are recorded simultaneously and from these data the natural frequencies and mode shapes are identified. In some cases snapback tests and impulse tests are used. When large structures are tested, the large excitation force may result in local structural damage during testing.

Recently, analysis of the background vibration has been used to predict the structural dynamic properties of bridges and tall buildings. This study explores the feasibility of using background vibration measurements in a CANDU plant to determine the structural dynamic properties of large equipment.

Typical vibration measurements from various equipment in a CANDU plant have been studied. Analytical techniques have been developed to use the measured background vibration data to predict mode shapes and natural frequencies. The accuracy of the analytical techniques have been tested on an example cantilever beam with known dynamic characteristics. The equipment necessary to apply this technique in a CANDU plant has been identified and an experiment to demonstrate the technique at the laboratory stage has been designed.

The results of the study are presented in the main body of the report. Field measurement data, details of the analytical techniques and the listing of computer programs is presented in the appendices.

B. LITERATURE REVIEW

A literature survey was conducted to review the applications of background vibrations in the prediction of dynamic properties of structures. Most of the applications have been for bridges and tall structures. Wind-induced vibration has been measured and analyzed for predicting the structural dynamic properties of structures.[Ref. 2, 3, 4 and 5]

Wind pressure acting on the structure is considered as a stationary ergotic stochastic process known as a band-limited white noise with uniform power spectral density within the specified frequency range. The natural frequencies and mode shapes are determined from the power spectral density of the structural response. Another approach based on using environmental noise as a source of excitation was presented by E. Luz, et al. [Ref. 6] Structural response was recorded at various locations in a nuclear power plant, and the exciting force was considered as a white noise containing all frequencies with the same intensity. The authors successfully identified several natural frequencies and mode shapes within the frequency range of 0 to 16 Hz.

An interesting method for identification of structural dynamic properties was presented by V. R. McLamore, et al. [Ref. 7] The response of a suspension bridge to traffic, wind and other environmental factors was measured with sensitive seismometers. The recorded motion was analyzed using spectral techniques. The direct power spectral density of each

recorded motion yields the estimates of natural frequencies and modal damping, and the cross-spectral density between one of the measured responses considered as reference motion and all other recorded motions yields and the mode shape estimates.

The significant difference between this method and the one presented by E. Luz, et al [Ref. 6] is that the method used by V. R. McLamore, et al, relates the structural motion at various points to an arbitrary reference point on the bridge. This allows the identification of all modal characteristics, i.e., natural frequencies, damping ratios, and the phase differences between modes without any knowledge of the location of source excitation.

C. PLANT VIBRATION MEASUREMENTS

1. General

Plant vibration measurement data were obtained from Ontario Hydro Research Laboratory which conducts routine tests at the several nuclear, thermal and hydroelectric plants of the utility.[Ref, 8, 9, 10, 11 and 12]

The measured vibration data of typical elements of nuclear power plants were analyzed to estimate the range of accelerations, velocities and displacements one can expect in a CANDU plant.

Most of the measurements have been conducted to analyze fatigue failures caused by vibrations within the range of frequencies from 2 kHz to 20 kHz. The transudcers used in such cases were not sensitive enough to detect lower frequencies below 50 Hz which are of interest in earthquake response analysis. Therefore, such data were not considered useful for this study. However, several data records contain response in the range of frequencies from 0 Hz to 50 Hz and these data are presented in the following sections. Details of the measurements and data are presented in Appendix A.

2. Liquid Zone Control Unit

At the Bruce 'A' Nuclear Generating Station, a vibration monitoring study has been conducted to investigate the Liquid Zone Control Unit (LZCU)

failure. The measurements of reactivity control deck displacements and velocities versus liquid zone control unit displacements and velocities (Appendix A) show a coherence of deck motion and the LZCU motion. The y-direction of motion of the LZCU and the deck are coherent and in phase from 3 Hz to 7 Hz. A similar range of frequencies exists for the x-direction. The frequency range of the deck motion measurements is from 0 Hz to 20 Hz which is the range of interest for seismic analysis. The recorded displacements are 6 μm to 20 μm. The reactor was at 92% power with all systems in operation. The measured spectrum at the deck shows that the background vibration is a combination of random and discrete vibrations. These data could be used to identify the mode shapes and natural frequencies of the reactivity control deck and LZCU system.

3. Feeder Cabinet

At the Pickering Nuclear generating station vibrations of the feeder cabinets were monitored (Appendix A). Although the recorded vibration spectrum ranges from 0 to 20 kHz, it shows the presence of random and discrete vibrations in the lower frequency range.

4. Feeder Pipe

At the Pickering Nuclear generating station, vibrations of the feeder pipe were monitored. The spectrum of the data shows a combination of random and discrete vibrations. The maximum feeder pipe acceleration measured under full load conditions is 1.25 g within the range of frequency from 0 Hz to 50 Hz, the maximum velocity 12.7 mm/s and maximum displacement 1.35 mm.

5. Feedwater Heater

At the Thunder Bay Thermal generating station vibrations of the feedwater heater were monitored (Appendix A). The measurements indicates random vibration in the low frequency range. The maximum velocity of the support is 0.553 mm/s and the maximum acceleration 0.023 g.

The following table gives a summary of the maximum acceleration, velocities and displacements at various locations discussed above. The frequency range is from 0 Hz to 50 Hz.

Location	Acceler-ation (g)	Velocity (mm/s)	Displacements (mm)	Transducers
Bruce Nuclear Generating Station A	measured	integrated acceleration	double integrated acceleration	B&K 4368 B&K 2635
Reactivity Control Deck and LZCU	from 0.38 to 0.58	from 0.55 to 2.0	from 0.006 to 0.020	
Pickering Nuclear Generating Station	measured	not known	not known	not known
Feeder Pipe	0.5	12.7	1.35	
Guide Tube	1.25	12.7	0.55	
Thunder Bay Generating Station Unit 2	measured	integrated acceleration		B&K 4368 B&K 2635
Feedwater Heater	0.023	0.553		

D. METHOD OF ANALYSIS

A summary of the analytical method used for the identification of the dynamic charcteristics of a structure is presented in this section. The objective of the method is to identify the natural frequencies and corresponding mode shapes of the structure (Figure 1) where the nature of the excitations is not precisely known. The motion of the structure is governed by system of coupled equations.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \qquad (D.1)$$

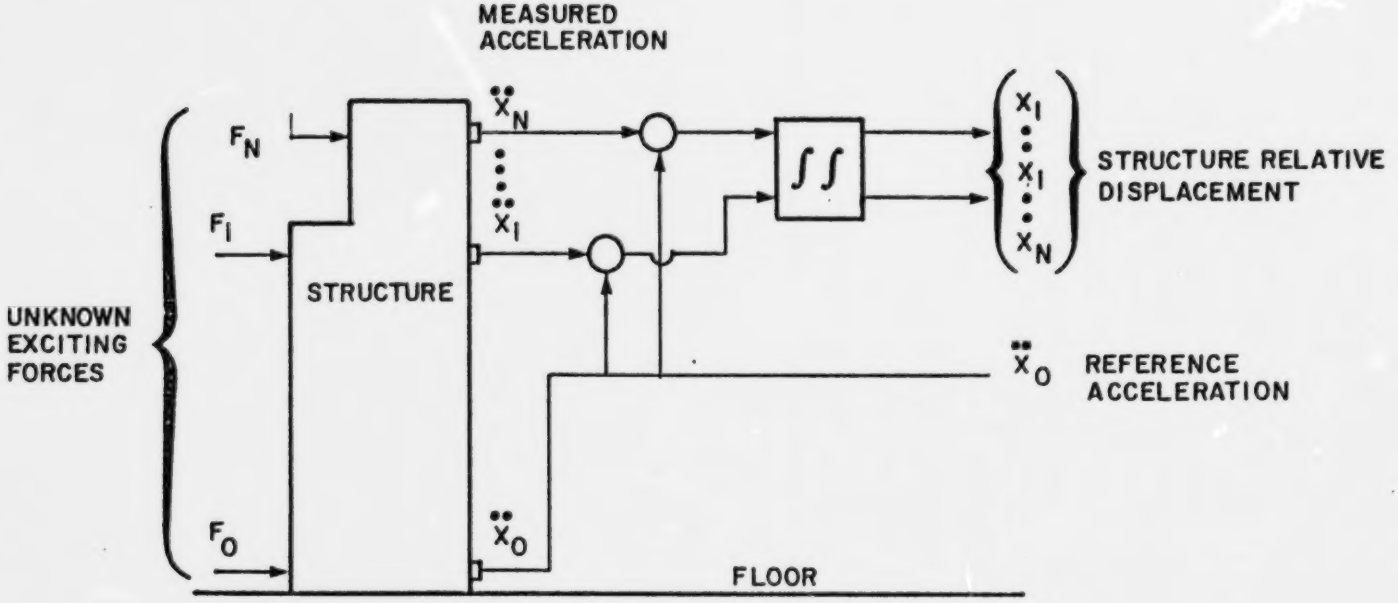


FIG. 1

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

STRUCTURE MOTION

ACRES

where

$\{\ddot{x}\}$, $\{\dot{x}\}$, $\{x\}$ = vector of structure nodal acceleration, velocity and displacement, respectively

$[M]$ = mass matrix

For identification purposes it is desirable to extract from the records of motion the structural response to ambient excitations. Therefore, the structural motion has to be referred to some reference point $x_o(t)$ on the structure. All other motions $x(t)$ are expressed in terms of relative coordinates $x_r(t) = x(t) - x_o(t)$. The structure equation of motion after rearrangement now becomes

$$[M^1]\ \{\ddot{x}_r\} + [C^1]\ \{\dot{x}_r\} + [K^1]\ \{x_r\} = \{F^1\} \qquad (D.2)$$

The new matrices $[M^1]$, $[C^1]$ and $[K^1]$ are obtained from the old ones (Equation D.1) by deleting the row and column corresponding to the reference point $x_o(t)$. The new force vector $\{F^1\}$ contains the old elements F_1, F_2, F_N, and the new ones as a result of transformation

$$F^1{}_i = F_i - m_i\ddot{x}_o - c_i\dot{x}_o - k_i x_o \qquad (D.3)$$

where

m_i = mass associated with the transducer i

c_i = sum of the elements of the matrix [C] in the i-th row

k_i = sum of the elements of the matrix [K] in the i-th row.

The values c_i and k_i are small compared to the diagonal elements of the matrix [C] and [K] and it is a common practice in mode shape and frequency analysis to neglect the static component $-k_i x_o$ and the viscous component $-c_i\dot{x}_o$ as small compared to the dynamic component $-m_i\ddot{x}_o$.

Formally, the Equation D.2 is the same as Equation D.1 (and the Equation B.1.1 in Appendix B) and the term $-[M^1]\ddot{x}_o$ can be considered as an additional exciting force. Further assumption has been made in that the structure does not have close natural frequencies and each mode can be approximately considered as a single degree of freedom in the vicinity of the resonant frequency.

The system response in the frequency domain is

$$\{x_r(f)\} = [H(f)]\ \{F - [M]\ \ddot{x}_o\} \qquad (D.4)$$

where

$\{x_r(f)\}$ = spectrum of output displacements

$[H(f)]$ = frequency response function

$\{F\}$ = vector of unknown exciting forces

$[M]$ = is a reduced mass matrix where the superscript is being dropped for convenience

$\ddot{x}_o(f)$ = spectrum of reference acceleration

f = frequency (Hz)

The modal parameters corresponding to the natural frequency, f_r are determined from cross-spectral density function $S_{\ddot{x}_o x_r}(f)$.

The function $S_{\ddot{x}_o x_i}(f)$ relates the reference acceleration $\ddot{x}_o(t)$ to output displacement $x_i(t)$ in the frequency domain. Because the accelerations $\ddot{x}(t)$ are measured, to obtain the relative displacements $x_r(t)$ the double integration has to be performed over the difference between the measured acceleration records and the reference

acceleration $\ddot{x}_o$. It is assumed that the transducers are distributed uniformly on the structure so that the mass associated with each transducer is the same and equal to m_o. The structural relative response $x_r(t)$ also contains the contribution from unknown exciting force vector $\{F\}$. The reference acceleration $\ddot{x}_o(t)$ and the exciting forces $\{F\}$ are generally uncorrelated so the effect of $\{F\}$ on reference motion $\{x_r(t)\}$ can be considered as noise.

The frequency response function at node i is computed from the relation (Appendix B)

$$H_{io}(f) = \frac{S_{\ddot{x}_o x_i}(f)}{m_i S_{\ddot{x}_o \ddot{x}_o}(f)} \qquad (D.5)$$

where the index r has been replaced by i and

$S_{\ddot{x}_o x_i}(f)$ = cross-spectral density function of output x_i with respect to input $\ddot{x}_o$

$S_{\ddot{x}_o \ddot{x}_o}(f)$ = spectral density function of the reference otion $\ddot{x}_o$.

$H_{io}(f)$ = frequency response function at the point i

$m_i = m_o$ = mass of structure associated with node i

The frequency response function $H_{io}(w)$ is scaled by the unknown mass m_o. This scale factor vanishes during the mode normalization procedure where the first element of the eigenvector is set to unity. The frequency response function (FRF) obtained from relation (D.5) is a complex variable and contains information about the amplitude of the displacement and the phase angle at node i. The velocity of the chosen reference point can be evaluated by checking the coherence function $\gamma^2_{\ddot{x}_o x_i}$ between the reference acceleration $\ddot{x}_o$ and the relative displacement x_i

$$\gamma^2_{\bar{x}_o x_i} = \frac{|S_{\bar{x}_o x_i}|^2}{S_{\bar{x}_o \bar{x}_o} S_{x_i x_i}}, \quad 0 \leq \gamma \leq 1 \tag{D.6}$$

Good correlation between $\bar{x}_o$ and x_i is observed when the value of γ approaches unity. In case the coherence function is small (γ close to 0) then another reference point has to be selected. A detailed description of modal parameter identification from the frequency response function estimate is demonstrated in Appendix B.

E. EVALUATION OF MODAL CHARACTERISTICS OF A CANTILEVER BEAM

To verify the method of analysis the example of a simple cantilever beam excited by white noise was studied. The natural frequencies and mode shapes of a cantilever beam (Figure 2) subjected to excitations caused by floor motion were evaluated. The flowchart shown in Figure 3 illustrates the verification procedure. In the first part of the verifiction procedure the response of the cantilever beam to random excitation was calculated. The method of analysis described in Chapter D was then used to predict the modal characteristics from the calculated responses and compared with those calculated directly from the structural properties. The elastic beam response (displacements) at four locations (nodes) to a randomly excited floor motion was obtained using the structural analysis computer program (SAP4). The floor motion was generated numerically (Figure 4) and its autocorrelation function (Figure 5.1) and spectral density showed that it could be considered as a white noise (Figure 6.1). SAP4 uses the structural properties of the beam to compute its natural frequencies and mode shapes. These modal properties are then used by the program to calculate the response displacements. The results obtained from SAP4 (Figure 7) were used to evaluate cross-spectral density functions for acceleration- displacement relations (Figure 5.2 and 6.2). Then the digital estimates of frequency response functions at four locations on the cantilever beam were obtained (Figures 8 to 15). To minimize the leakage error, the Hanning window [Ref. 13] function was implemented into the algorithm. The comparison of the two lowest natural frequencies and corresponding normalized mode shapes, determined from Figures 8 to 15 graphically, is shown in Table 1 and in Figure 16.

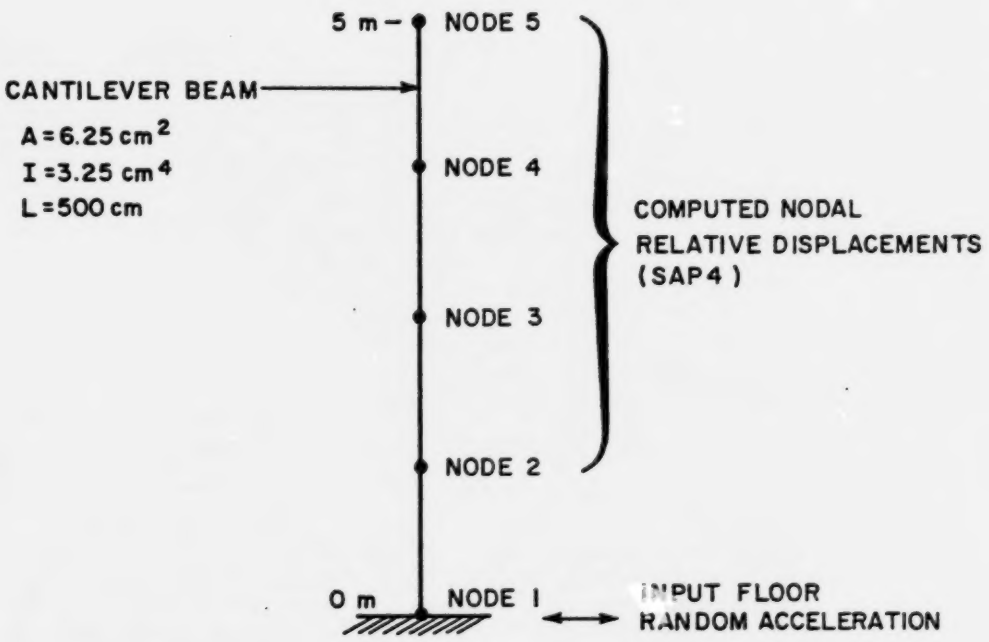


FIG. 2

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

FOUNDATION MOTION INPUT

ACRES

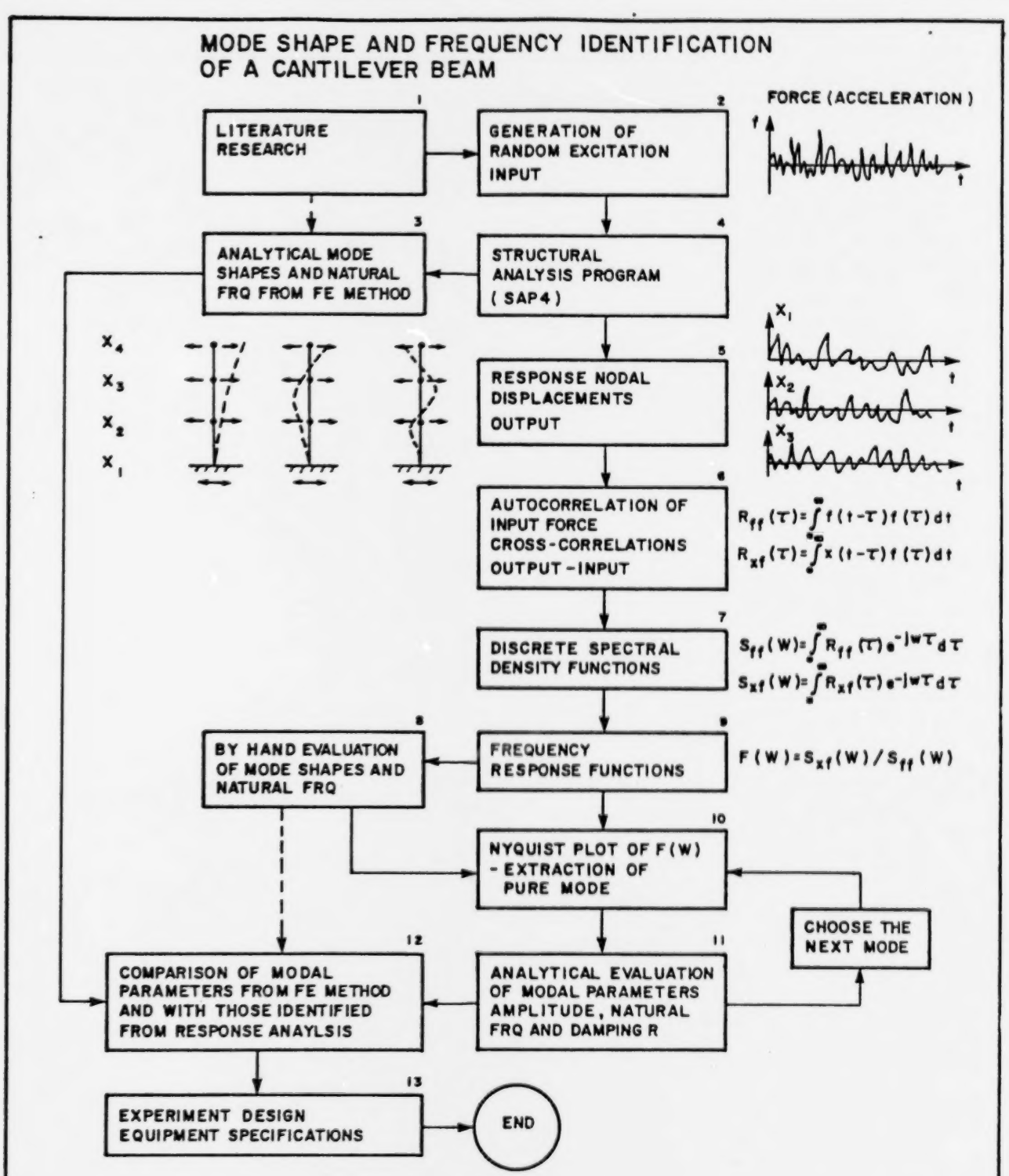


FIG. 3

THE FLOW CHART
OF IDENTIFICATION OF BEAM DYNAMIC CHARACTERISTICS

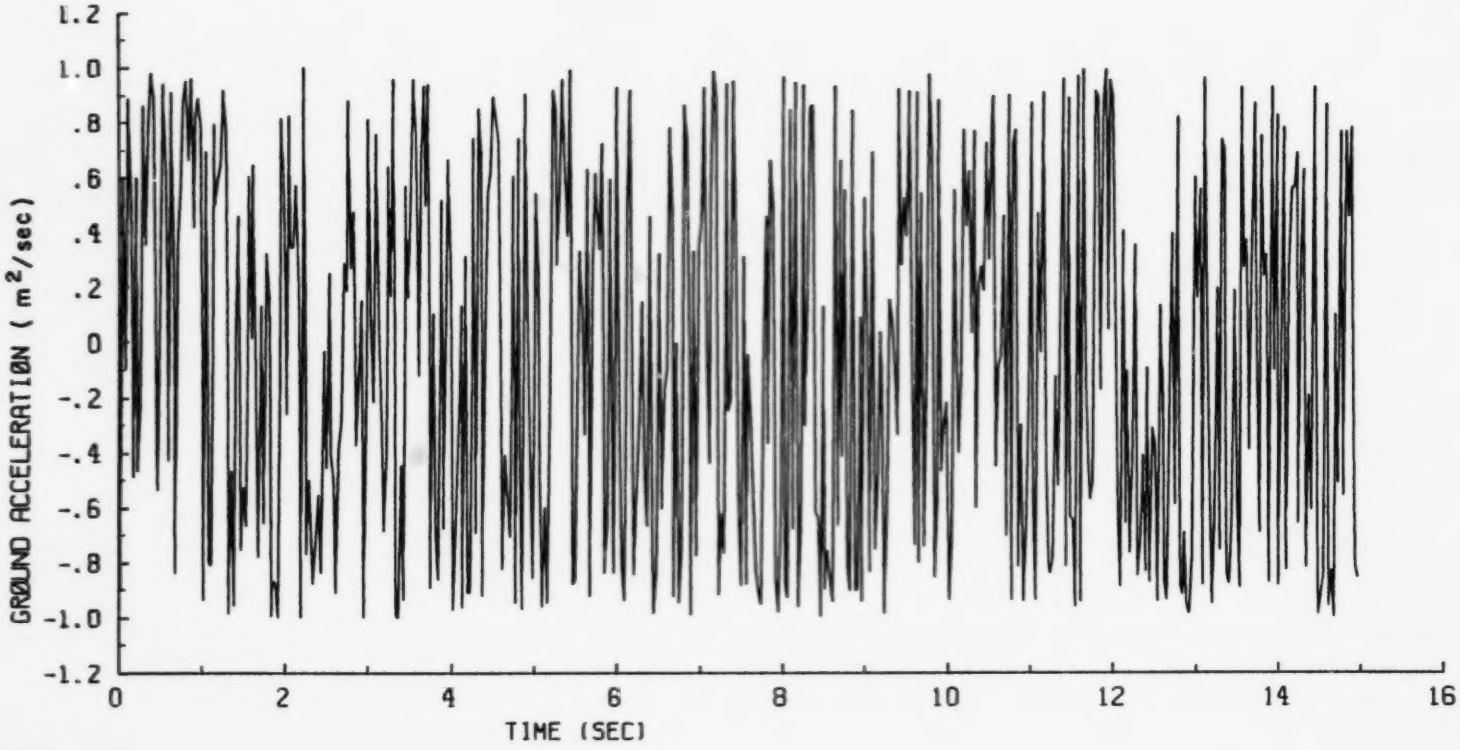


FIG. 4

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

GROUND ACCELERATION

ACRES

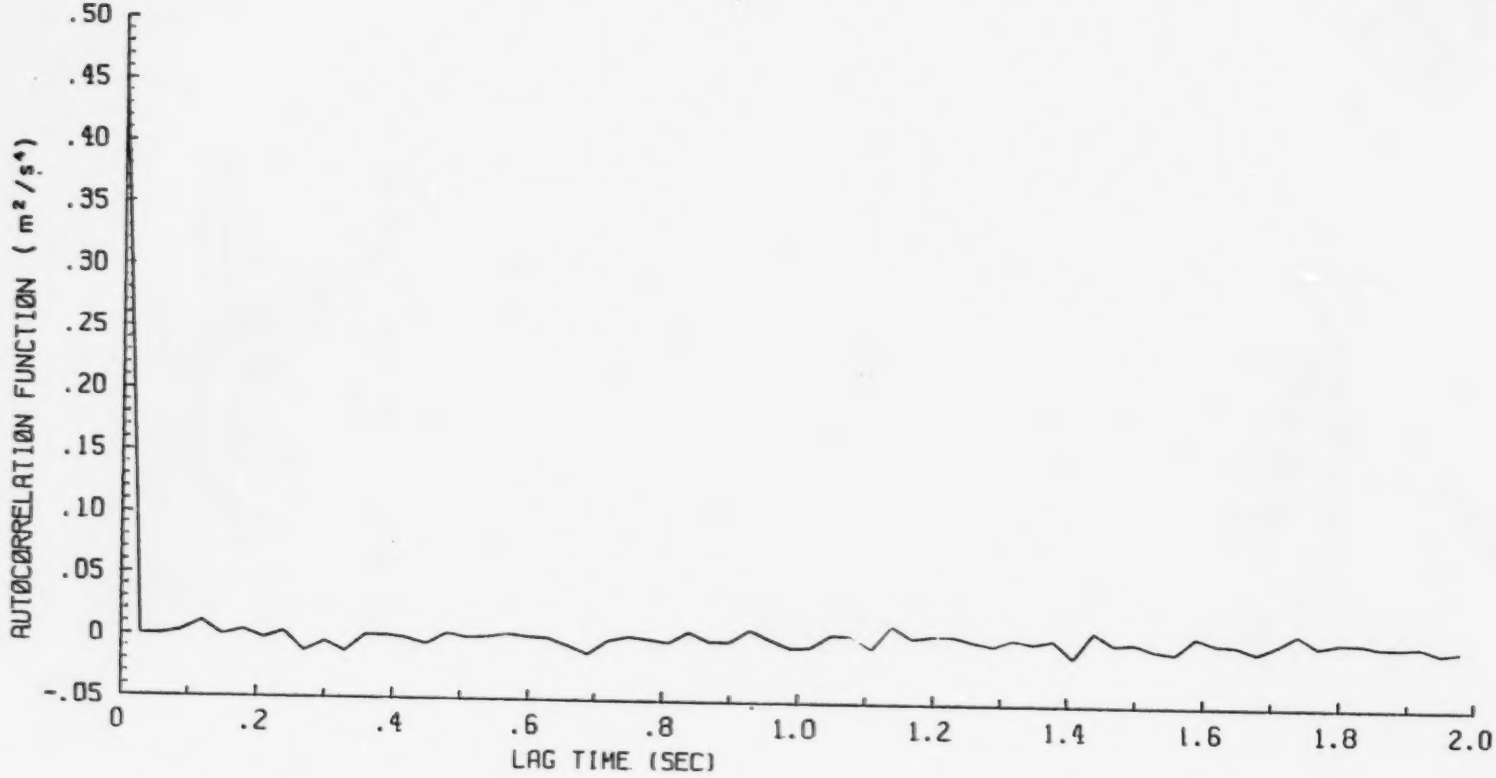


FIG 5.1

AECB

MØDE SHAPE AND FREQUENCY

INPUT AUTØCØRRELATIØN FUNCTIØN

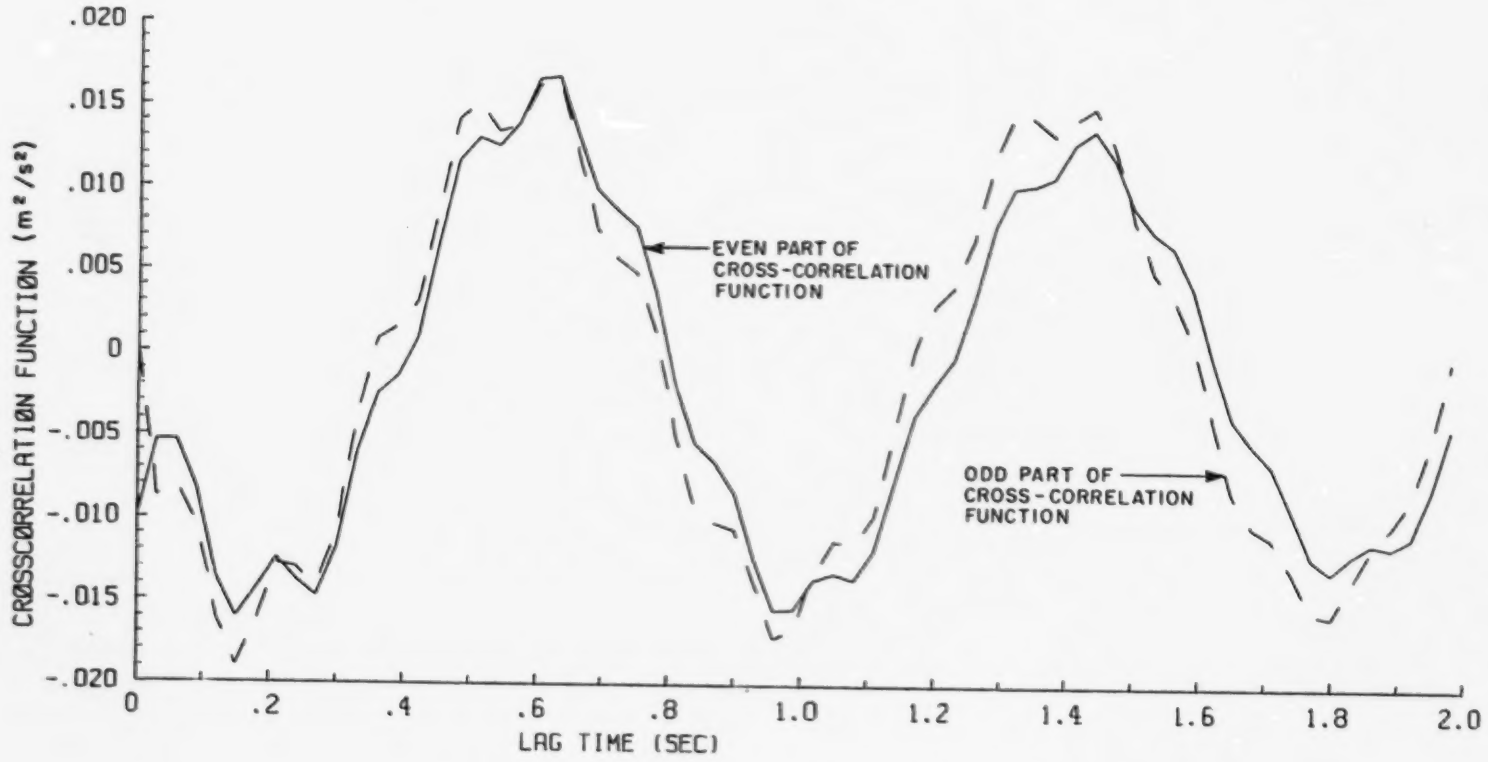


FIG 5.2

AECB

MØDE SHAPE AND FREQUENCY

ØUTPUT CRØSSCØRRELATIØN AT NØDE 2

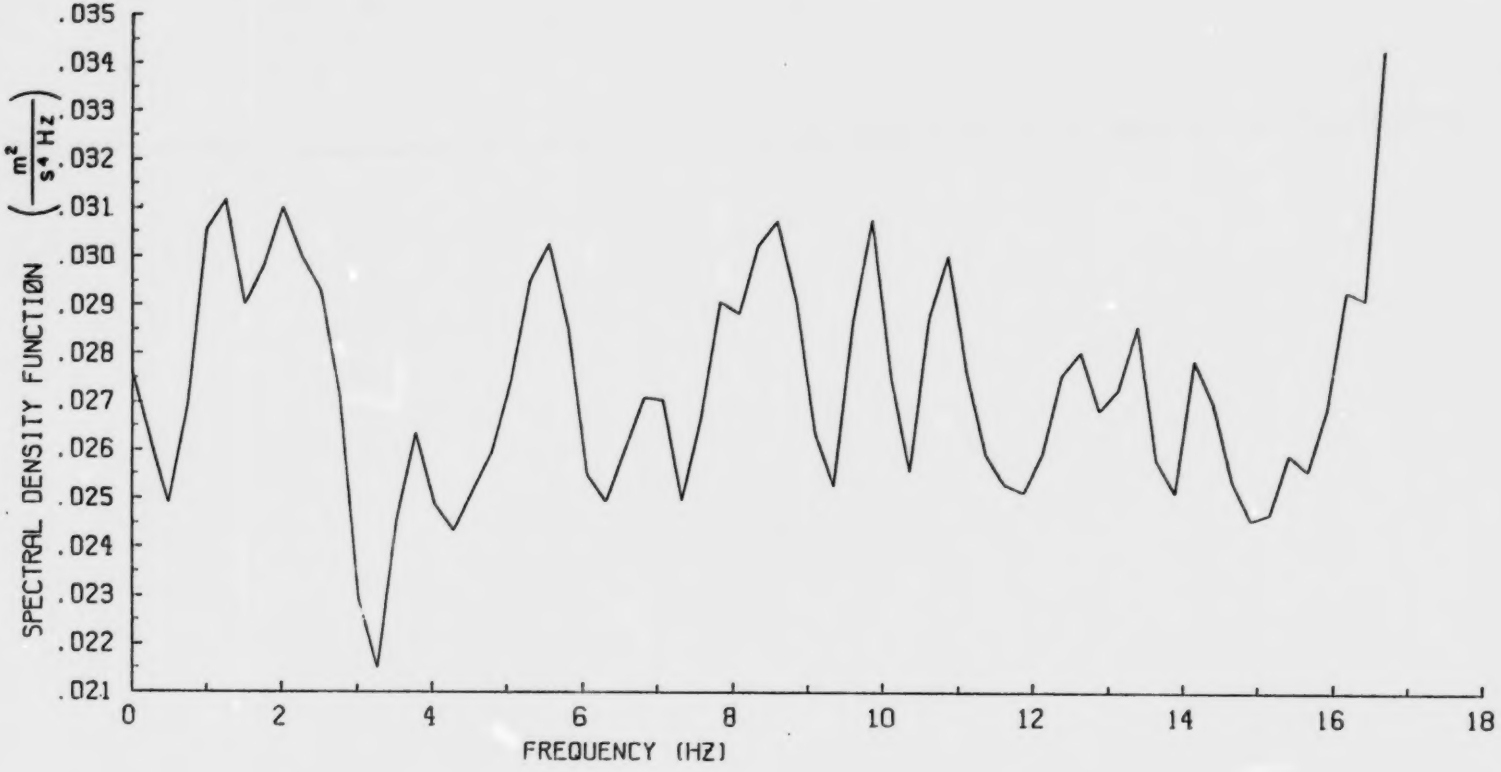


FIG 6.1

AECB

MØDE SHAPE AND FREQUENCY

SPECTRAL DENSITY ØF INPUT

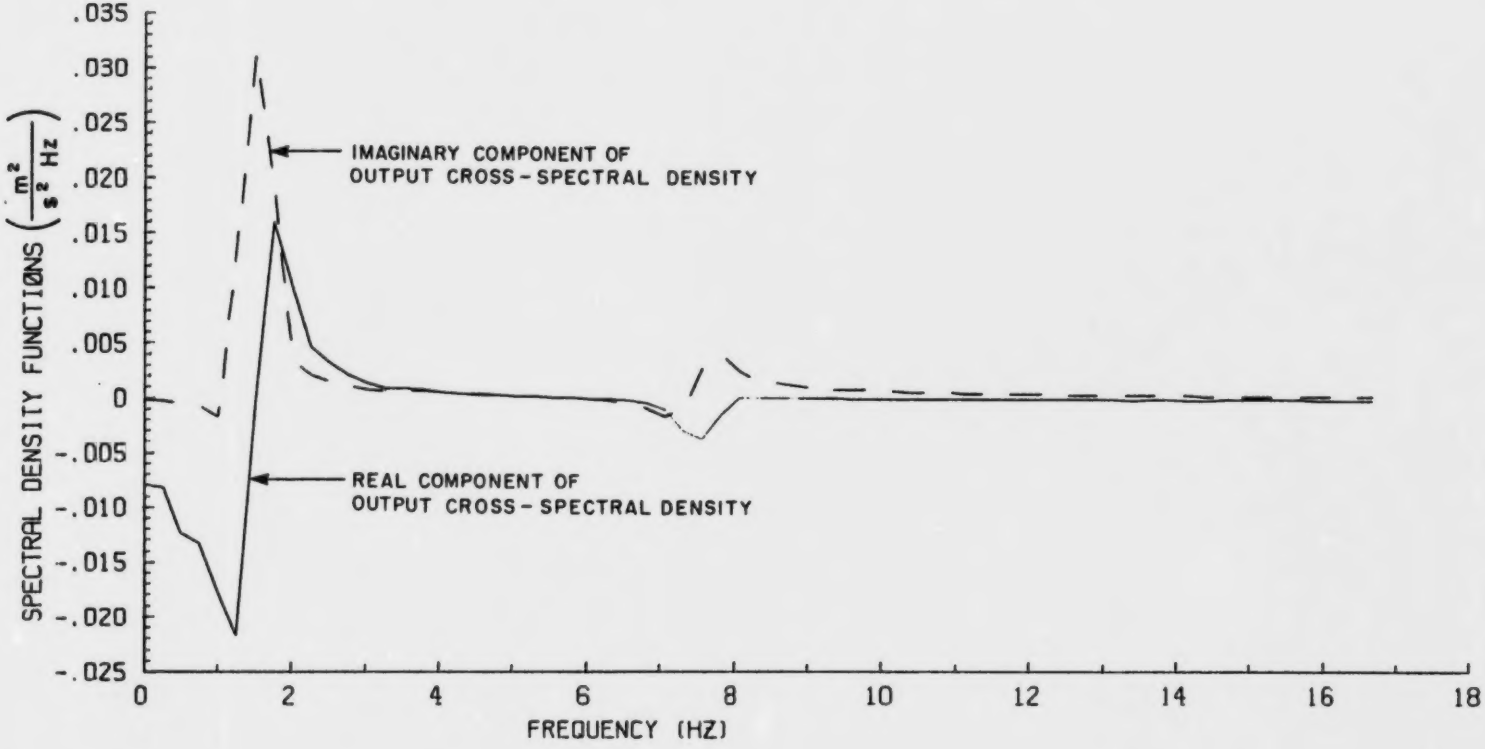


FIG 6.2

AECB

MODE SHAPE AND FREQUENCY

OUTPUT SPECTRAL DENSITY AT NODE 2

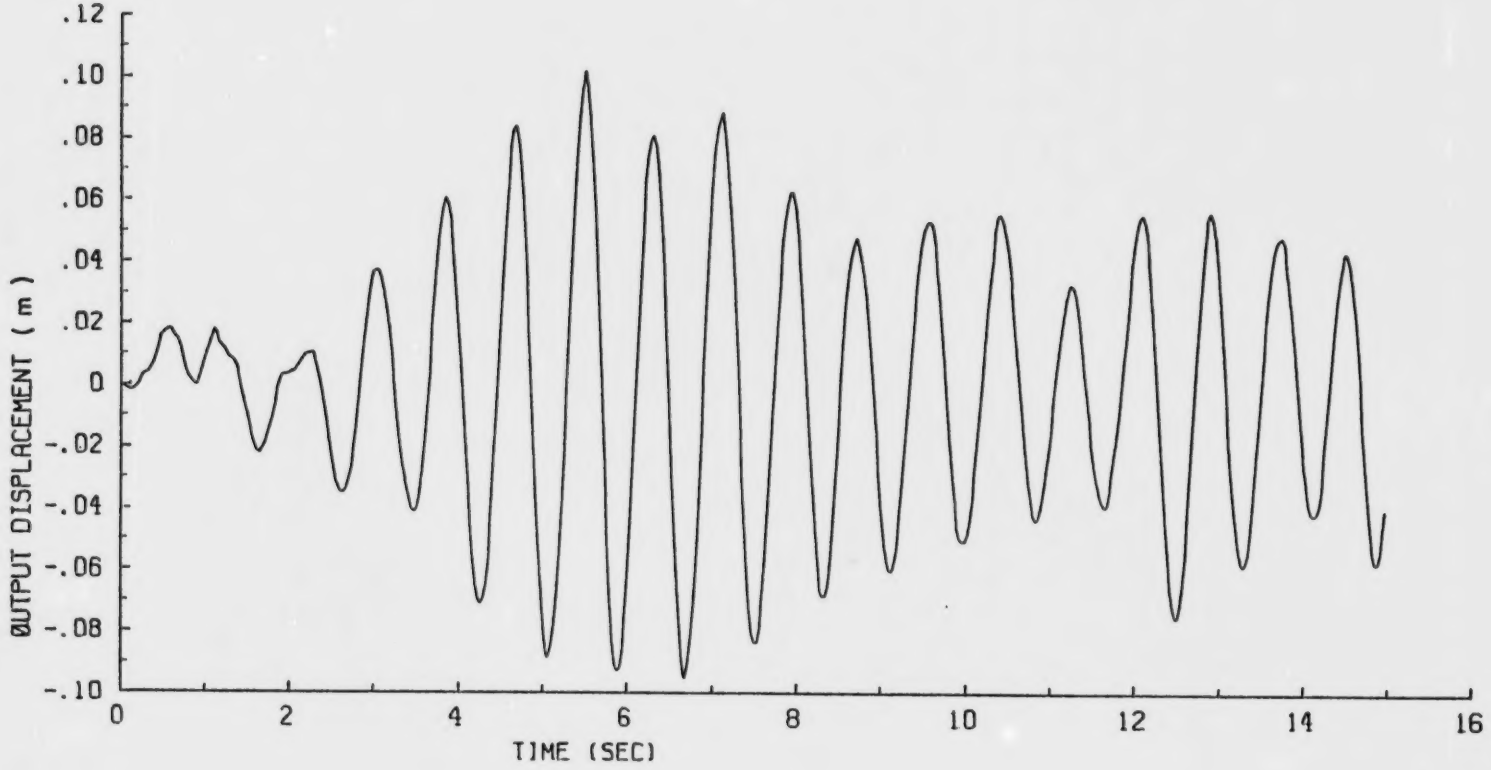


FIG. 7

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

BEAM DISPLACEMENT AT NODE 2

ACRES

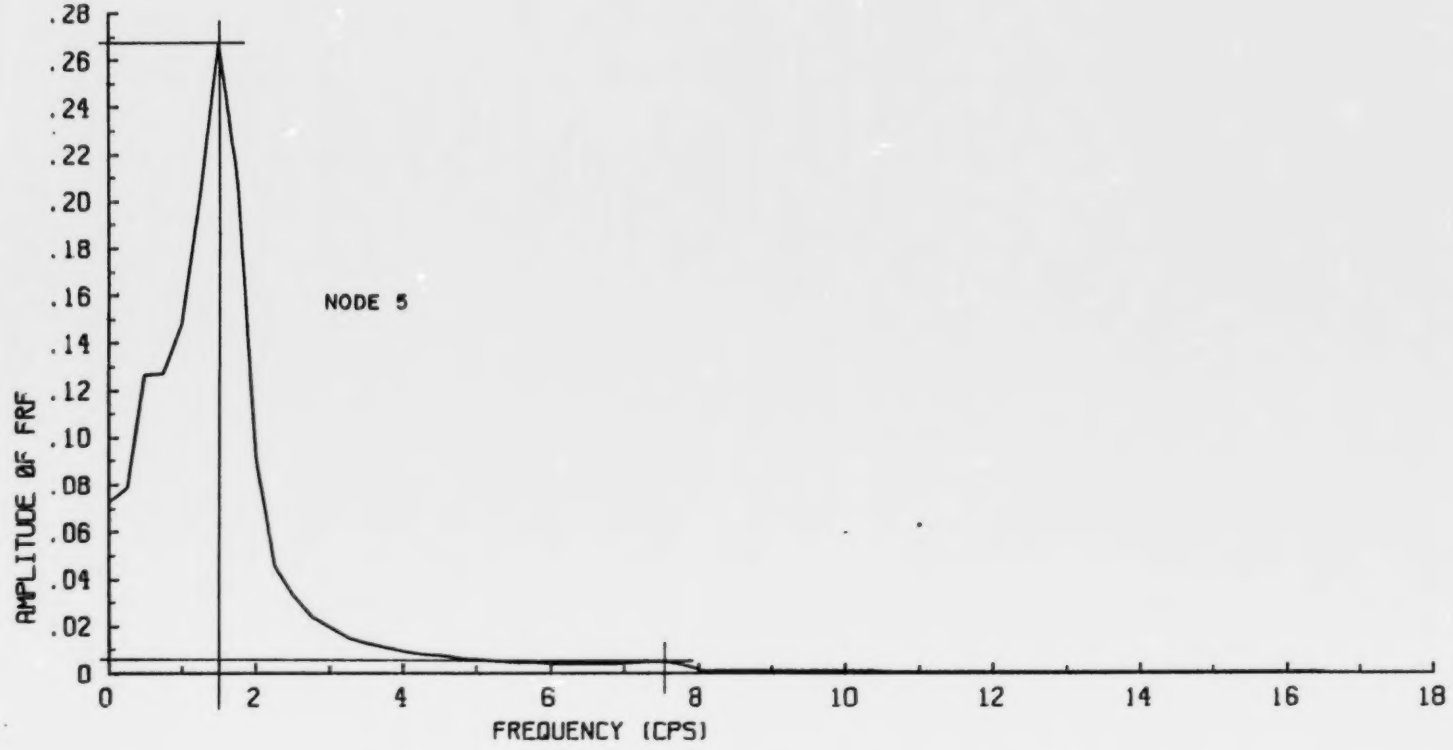


FIG.8

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

ACRES

AMPLITUDE OF FREQUENCY RESPONSE FUNCTION

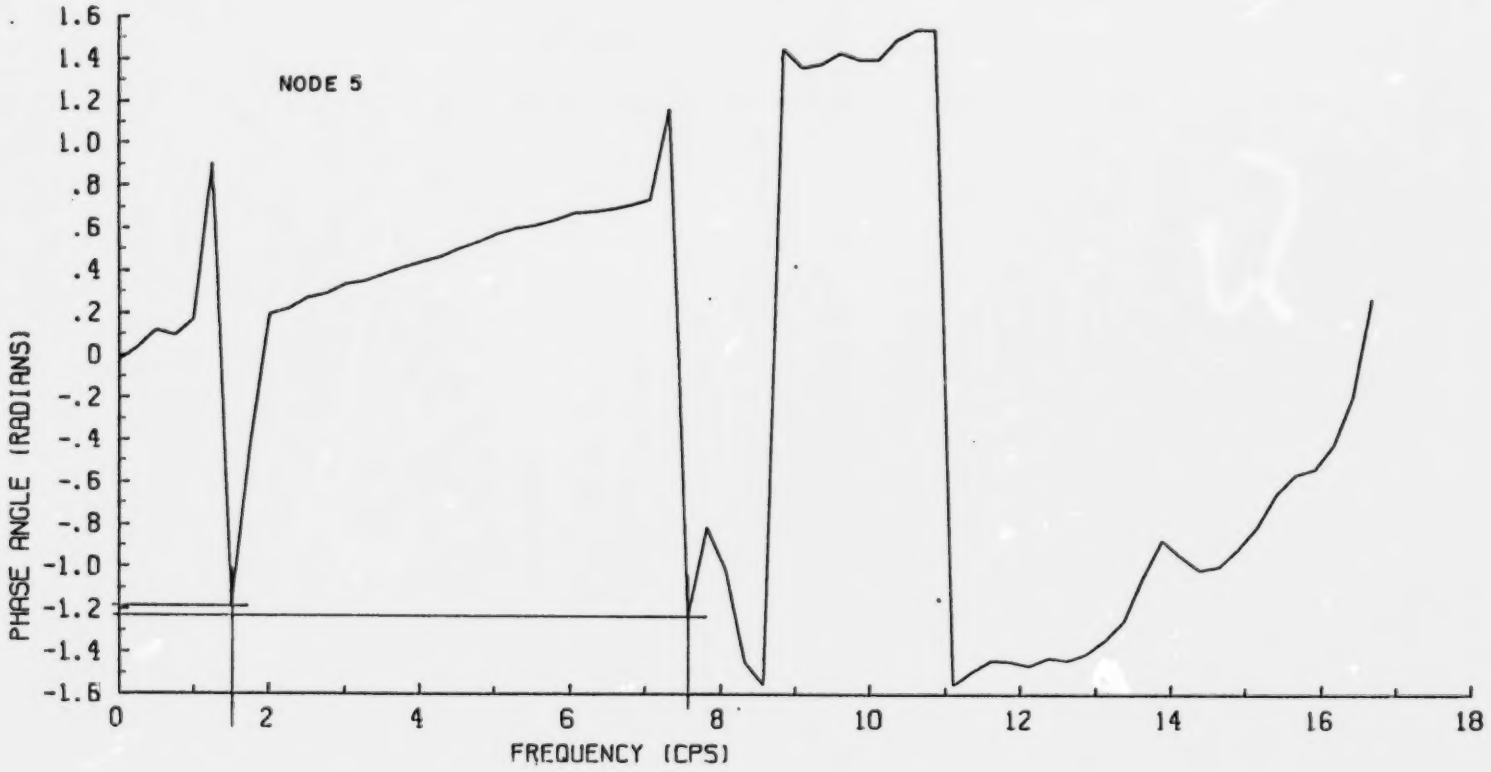


FIG. 9

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

PHASE OF FREQUENCY RESPONSE FUNCTION

ACRES

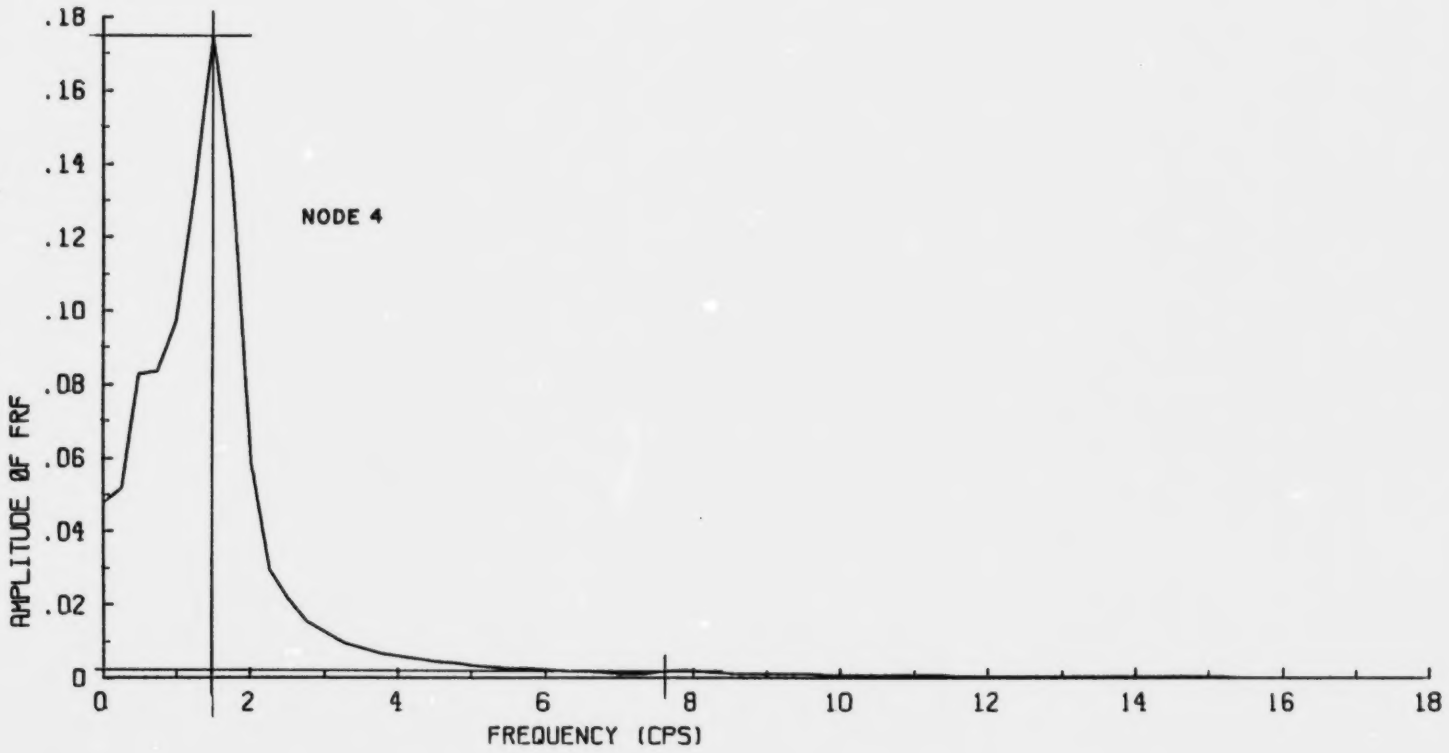


FIG. 10

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

AMPLITUDE OF FREQUENCY RESPONSE FUNCTION

ACRES

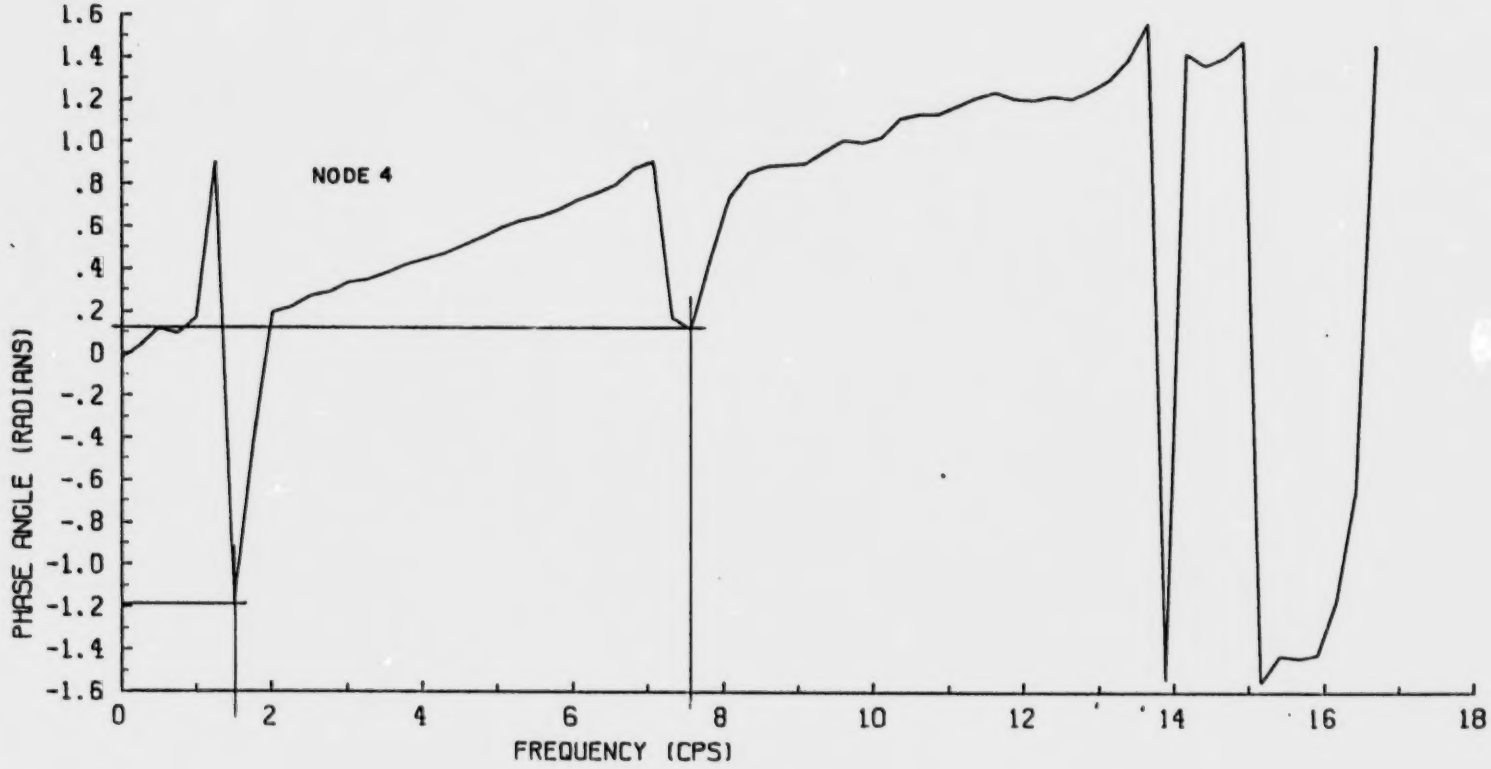


FIG. II

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

PHASE OF FREQUENCY RESPONSE FUNCTION

ACRES

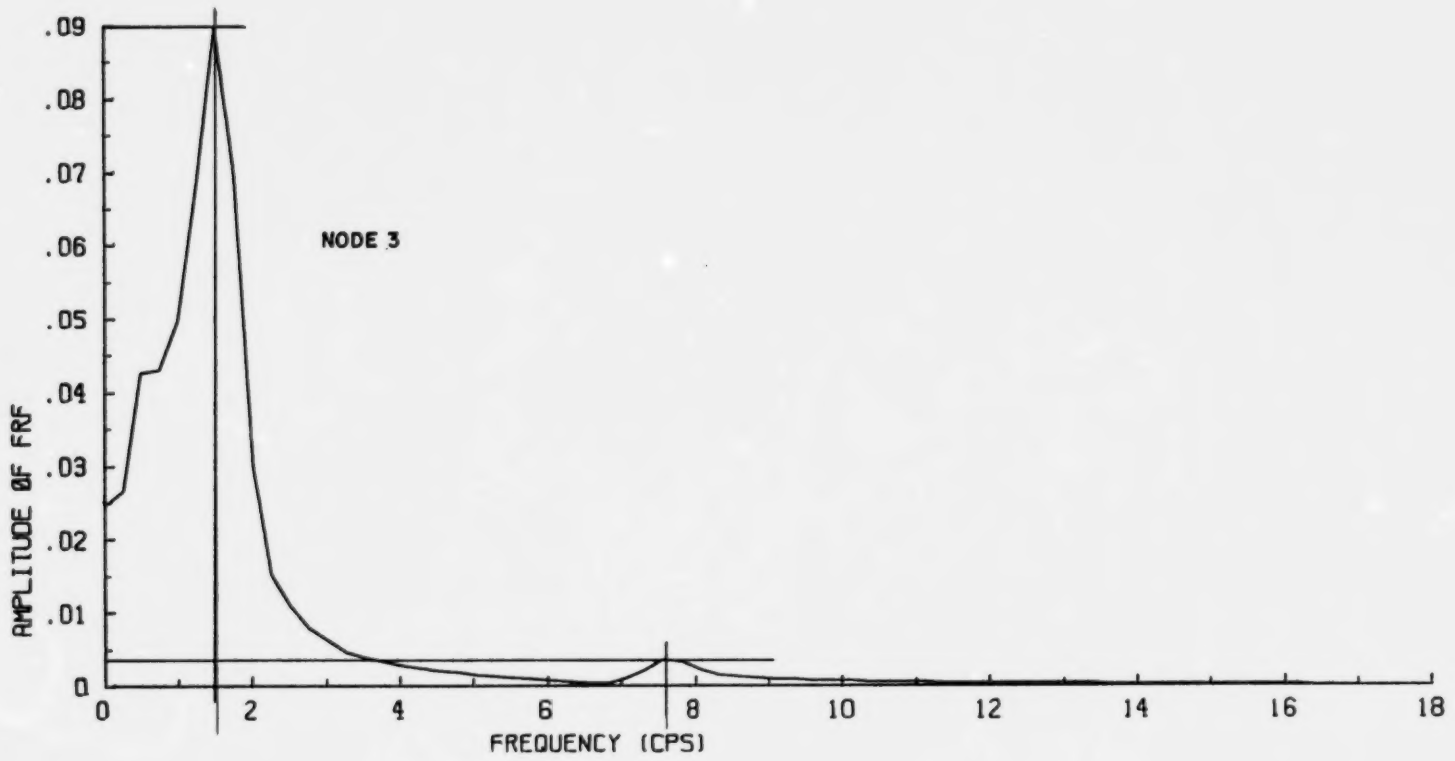


FIG. 12

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

AMPLITUDE OF FREQUENCY RESPONSE FUNCTION

ACRES

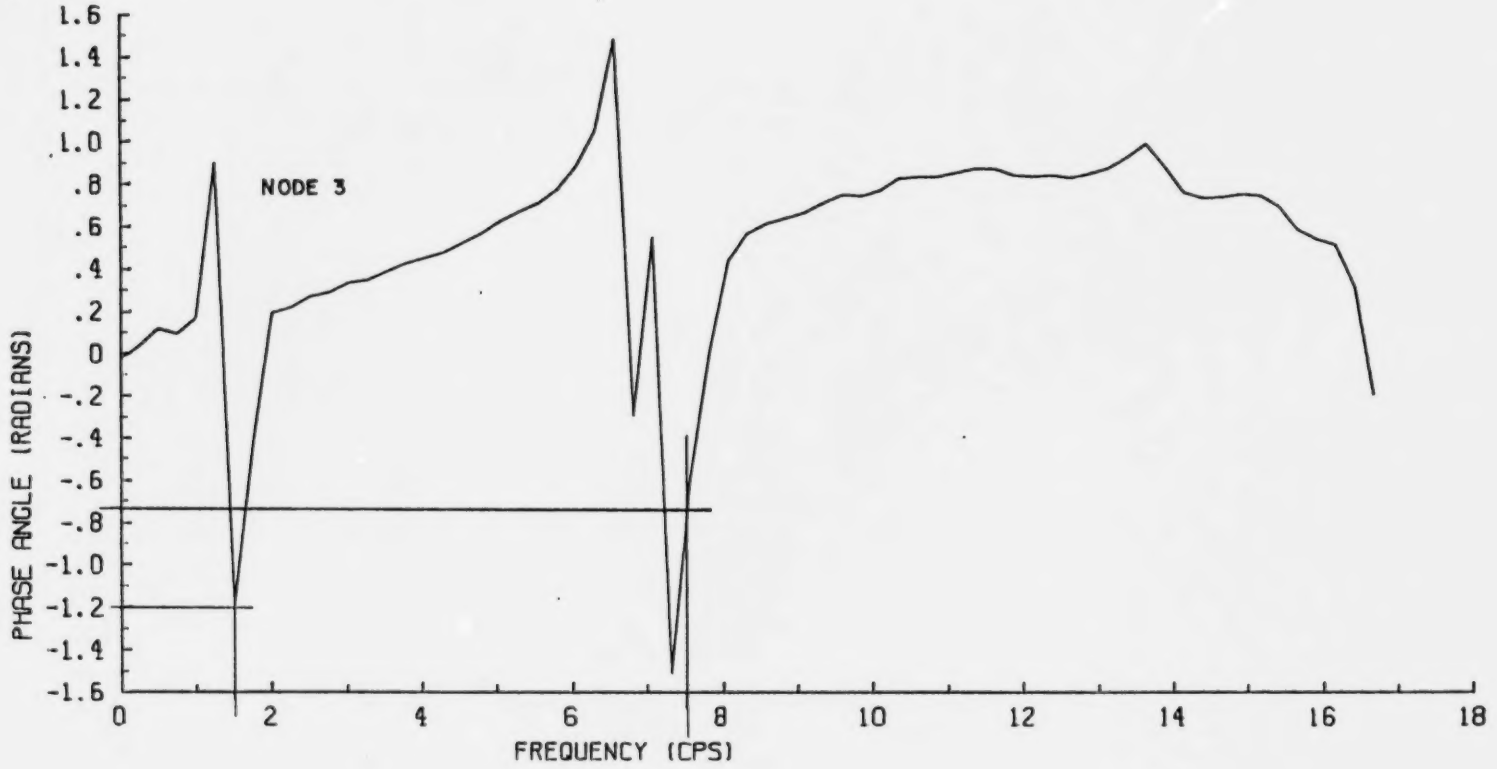


FIG. 13

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

PHASE OF FREQUENCY RESPONSE FUNCTION

ACRES

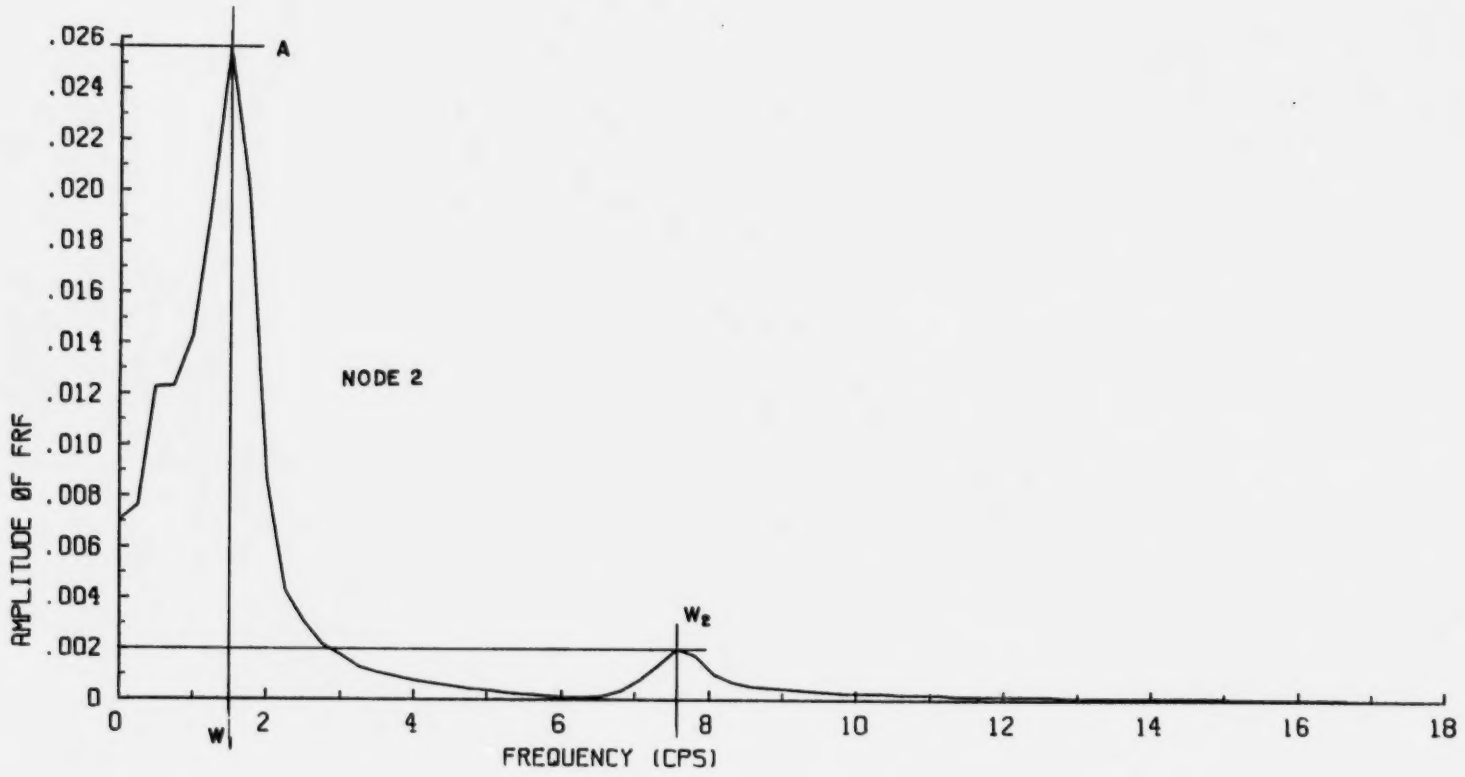


FIG. 14

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

AMPLITUDE OF FREQUENCY RESPONSE FUNCTION

ACRES

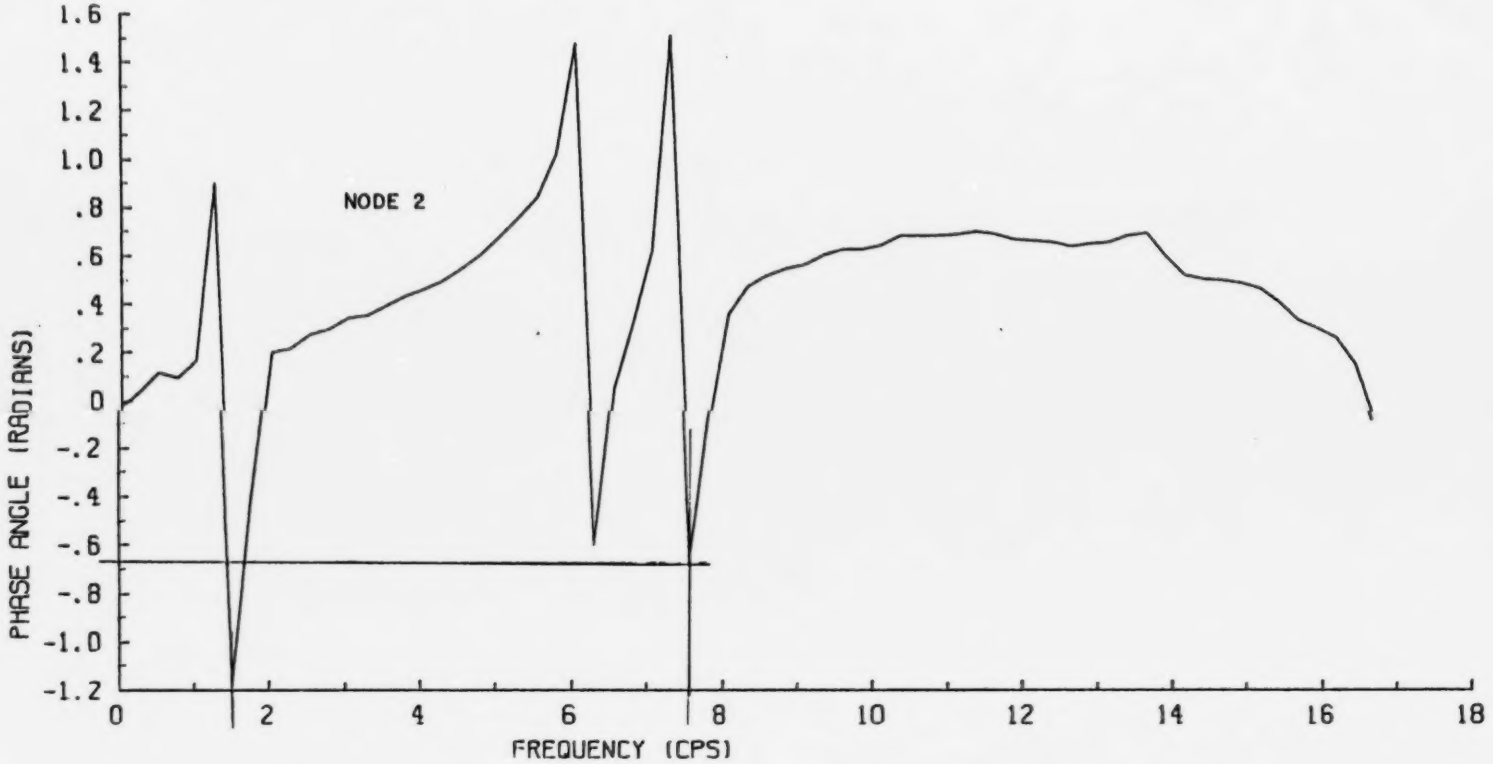


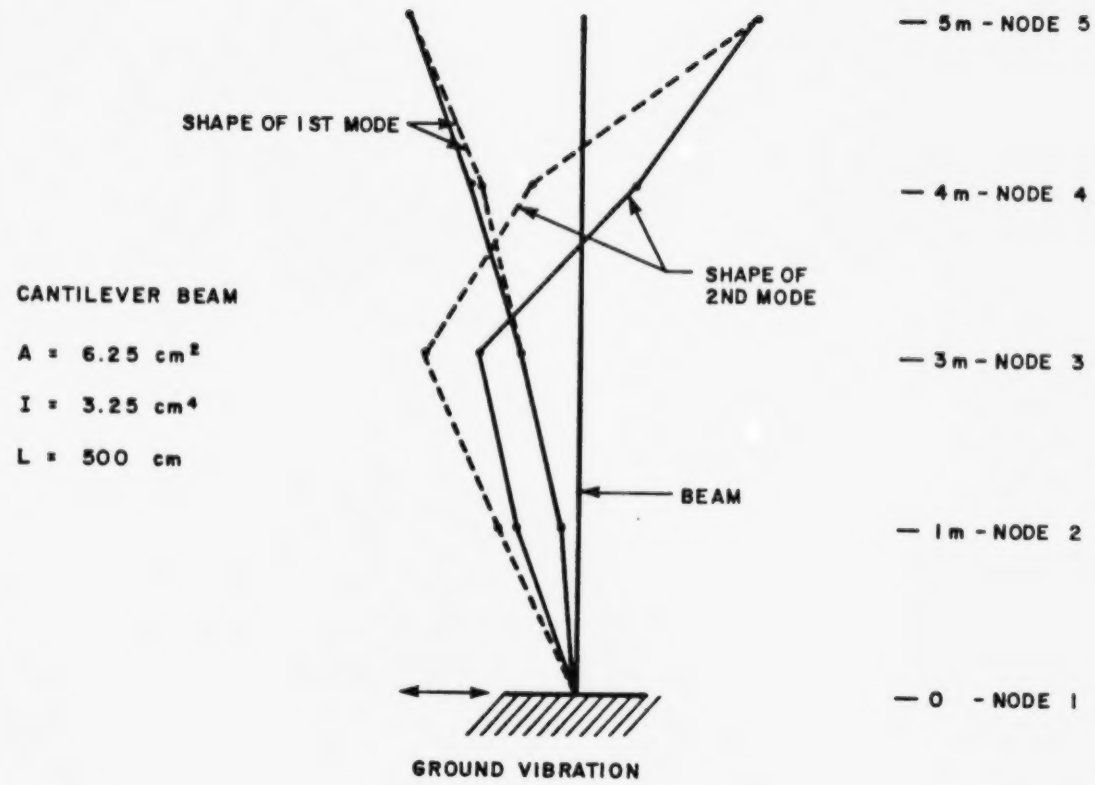
FIG. 15

ACEB

MODE SHAPES AND FREQUENCY IDENTIFICATION

PHASE OF FREQUENCY RESPONSE FUNCTION

ACRES



LEGEND

——— OBTAINED ESTIMATES

- - - - SAP4

FIG. 16

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

COMPARISON OF MODE SHAPES

ACRES

Table 1: Mode Shape and Natural Frequency Estimates of the Cantilever Beam

	MODE 1		MODE 2	
	Estimate	SAP4	Estimate	SAP4
NODE	f_1 = 1.5 Hz	f_1= 1.24 Hz	f_2= 7.6 Hz	f_2= 7.27 Hz
5	-1.0	-1.0	+1.0	+1.0
4	-0.67	-0.65	+0.35	-0.27
3	-0.34	-0.34	-0.69	-0.83
2	-0.10	-0.10	-0.38	-0.45
1	0.0	0.0	0.0	0.0

The mode corresponding to natural frequency f_1 = 1.24 Hz is well represented in measured output displacements and the results obtained from identifiction algorithm are close to those obtained from SAP4. The mode corresponding to natural frequency f_2 = 7.24 Hz was not activated sufficiently. The amplitudes of FRF along the beam are smaller than those corresponding to the first mode. Therefore, the second mode is strongly effected by truncation errors occurring during computation.

Another factor which effects the accuracy of obtained results is the digital representation of the structural motion. The required record length N, the number of lag values m, and the sampling interval h are related to the maximum anticipated frequency f_d, the rrequency bandwidth resolution B_e, and normalized standard error ε_r[Ref. 13]. For the analysed cantilever beam f_d = 7.5 Hz and B_e = 0.5 Hz. The standard error ε_r recommended for power spectrum estimate is ε_r = 0.1. Using relations obtained from Bendat, et al, [Ref. 13] the following parameters were determined.

The sample interval, $\Delta t = \frac{1}{4f_d} \approx 0.03$ s

The number of lag values, $m = \frac{1}{B_e \Delta t} = 67$

The sample size, $N = \frac{m}{\varepsilon_r^2} = 6700$

F. EQUIPMENT REQUIREMENTS

The equipment requirements for identifying the natural frequencies and mode shapes of major CANDU equipment, through application of the analytical techniques developed earlier, are discussed in this section.

The level of background vibration in a CANDU plant varies from one location to another. The equipment identified here may be used for measuring very low level vibrations in the 0 to 50-Hz range. On equipment where higher amplitude vibrations are present transducers with a higher amplitude range will be required. Figure 17 shows a conceptual arrangement of test equipment necessary to measure the response of a typical structure to background vibration and to analyze data for natural frequency and mode shape identification.

Shown on Table 2 are the brief specifications of typical equipment required for a field test and a lab experiment.

G. EXPERIMENT DESIGN

It is necessary to verify by experiment the proposed method of natural frequency and mode shape identification. Figure 18 shows the conceptual arrangement of such an experiment. The structure consists of a cantilever beam with two crossbars. Two adjustable weights are located on the top crossbar to excite and vary the torsional modes. The crossbar in the middle is used to mount accelerometers for the torsional mode identification. The cantilever structure will be attached rigidly to a heavy steel base. Several accelerometers will be mounted on the structure to measure horizontal accelerations in two orthogonal directions.

A vibration exciter run by a random function generator will be used to excite the structure with low level random noise at several locations which includes the base of the structure and the crossbars.

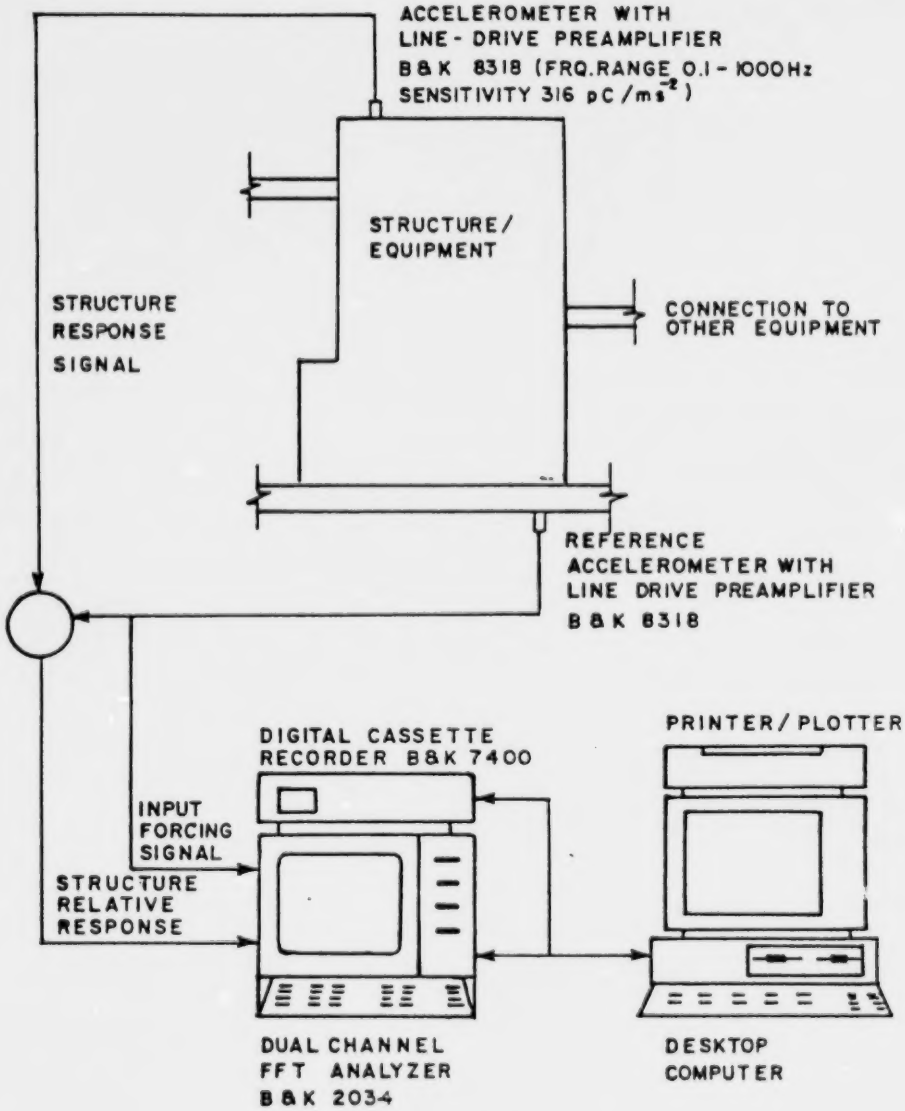


FIG. 17

AECB

BACKGROUND

MODE SHAPES AND FREQUENCY IDENTIFICATION

VIBRATION MEASUREMENT AND DATA ANALYSIS EQUIPMENT

ACRES

Table 2 Equipment Specification

	Equipment	Quantity	Cost $*	Equipment Specification
1	Dual channel FFT Analyzer B&K 2034	1	26,364	FIELD TEST
2	Digital Cassette Recorder B&K 7400	1	8,586	LABEXPERIMENT
3	Accelerometer B&K 8318	Min 4	1,439	
4	Vibration Exciter B&K 4809 (45N)	1	3,559	
5	Power Amplifier B&K 2706	1	2,635	

*Rent cost is 15% of purchase price per month.

The accelerometer data and the excitation signals will be recorded on a tape recorder. The recorded acceleration measurements will be digitized and analysed on a minicomputer (VAX 11/780). The analytical methods developed in this study will be used to predict the modal properties of the structure. Modal analysis techniques using harmonic excitation at several frequencies will also be used to measure the true natural frequencies of the structure. The natural frequencies and mode shapes calculated by the two methods will be compared.

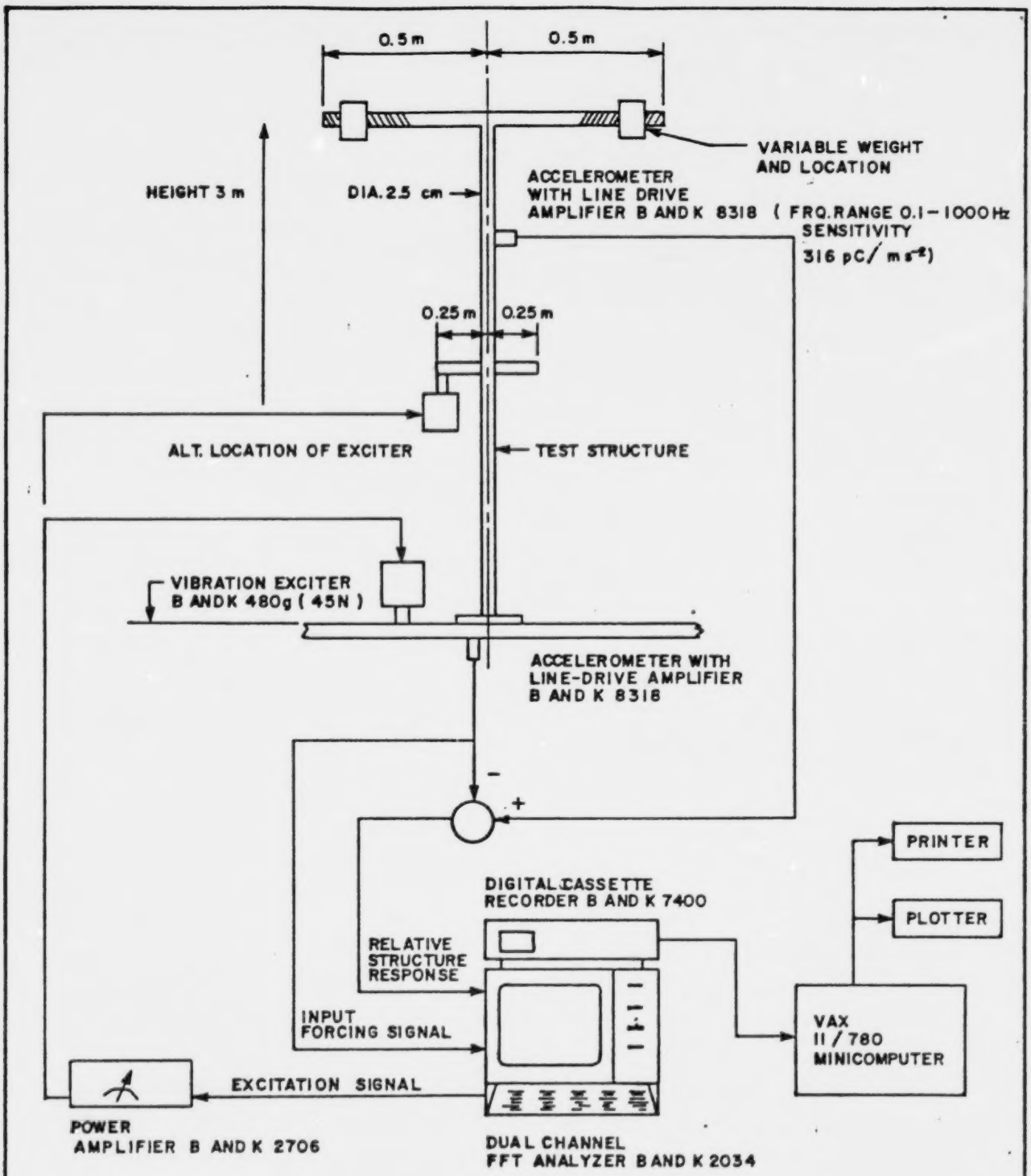


FIG. 18

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

LABORATORY TEST ARRANGEMENT

H. CONCLUSIONS

A review of the vibrations measurements conducted by Ontario Hydro indicates that there is measurable level of background vibration in the 0 to 50-Hz range present inside its CANDU plants. Although only limited data was available it is concluded that similar measurable background vibration must exist throughout the equipment in the plant. This vibration consists of random excitations approaching white noise and discrete harmonics.

The response of structures to random excitation has been studied and analytical methods have been developed which use measured response of structures to predict their dynamic characteristics.

The presented method of mode shape and natural frequency identification uses the ambient environmental noise as a source of excitation. The advantage of the method is that artificial excitation is not necessary, so that the measurements can be made during normal operation of the plant. Relating the structural motion to the measured reference motion allows the evaluation of amplitude and phase of the frequency response function without knowing precisely the input. The method is accurate provided that the mode is sufficiently activated. A comparison of the first two natural frequencies and the corresponding mode shapes of a cantilever beam to those obtained from the known dynamic properties by SAP4 program shows good agreement.

The accuracy of the method can be improved by implementation of the Kennedy-Pancu method [Ref. 1] or dynamic stiffness method [Ref. 15] for analytical evaluation of the modal characteristics. Further improvement in accuracy can be achieved by replacing the standard Fourier transformation with a Fast Fourier Transformation (FFT). The FFT significantly reduces number of arithmetic operations required to obtain the spectrum and thus, the total truncation error is much smaller.

In conclusion, it is feasible to use background vibrations in a CANDU plant to determine the dynamic characteristics of plant and structures for seismic analysis verification(s). However, further work is necessary to develop the analytical methods for improved accuracy of the dynamic characteristic prediction.

I. RECOMMENDATIONS

The feasibility of using response of structures and equipment in a CANDU plant to background vibration for predicting their dynamic characteristics has been proven in principle. However, further research needs to be conducted before the methods developed can be applied in the field. In this regard the following recommendations are made.

- Conduct further analytical studies to improve the accuracy of the methods for natural frequency and mode shape prediction. Apply the techniques to a number of different mathematical representation of structures to ensure that the methods are universally applicable.

- Conduct a laboratory experiment on a small-scale structure to verify the method of analysis.

- Apply the techniques so developed to a piece of equipment by conducting limited field tests in a CANDU plant.

The studies in the first two recommendations can be conducted simultaneously.

J. ACKNOWLEDGEMENT

The authors wish to acknowledge the cooperation of the Ontario Hydro Research Laboratory in supplying the vibration measurement data from its generating stations.

REFERENCES

[1] K. Zaveri, Modal Analysis of Large Structures - Multiple Exciter System, B&K, May 1984.

[2] H. S. Ward, H. H. Rainer, Experimental Determination of Structure and Foundation Parameters Using Wind-Induced Vibrations, The Institute of Civil Engineers, Vol. 53, pp. 305 - 321, September 1973.

[3] D. A. Foutch, E. Safak, Torsional Vibration of Along-Wind Excited Structures, Journal of the Engineering Mechanical Division, ASCE, Vol. 107, No. EM2, April 1981.

[4] G. C. Hart, Building Dynamics Due to Stochastic Wind Forces, Journal of Structural Division, Proceeding of the ASCE, pp. 535 - 550, March 1970.

[5] A. P. Jeary, P. E. Winney, Determination of Structural Damping of a Large Multiflue Chimney from the Response to Wind Excitation, The Institute of Civil Engineers, Vo.l 53, December 1972.

[6] E. Luz, C. Gurr-Beyer, W. Stoecklin, Identification of Natural Frequencies and Modes of a Nuclear Power Plant by Means of Excitation with Environment Noise. 7th International Conference on Structural Mechanics in Reactor Technology, Chicago, August 22-26, 1983, Vol. k(b), pp. 437 - 443.

[7] V. R. McLamore, G. C. Hart, I. R. Stubbs, Ambient Vibration of Two Suspension Bridges, Journal of Structural Divison, Proceedings of ASCE, pp. 2567 - 2562, October 1971.

[8] H. L. Anderson, Bruce NGS "A", Vibration Monitoring to Study Liquid Zone Control Unit Failure. Job 740627-118-551, File 825.4, dated April 8, 1983, Report No. 83-124-K.

[9] J. W. Forest, Feeder Pipe Vibration - Pickering GS. Job 740621-120, file 825.44, dated June 9, 1972, Report No. S72-49-K.

[10] Pickering NGS Feeder Cabinet Measurements performed June 7, 1982.

[11] Bruce NGS "B" On-Line Boiler Monitoring Unit 6 Boiler 3 and 4, February 1, 1985.

[12] R. F. Gibson, Thunder Bay GS Unit 5. Vibroacoustic Monitoring of of Feedwater Heater No. 5, Job 7406270-155-572, file 825.4, dated August 24, 1983, Report B83-55-K.

[13] J. S. Bendat, A. G. Piersol, Random Data: Analysis and Measurement Procedures, Wiley-Interscience, 1971.

[14] Harvard Vold, John Crowley and Thomas Rockin, New Ways of Estimating Frequency Response Functions, Sound and Vibration, pp. 34-38. November 1984, Volume 18, No. 11.

[15] B. J. Dobson, Modal Analysis Using Dynamic Stiffness Data, 3rd Intl. Modal Analysis Conference, Orlando, Florida, pp. 279-285. 1985.

BIBLIOGRAPHY

1. A. M. Abdel-Ghaffar, G. W. Housner, Ambient Vibration Tests of Suspension Bridge, Journal of the Engineering Mechanics Division, pp. 983 - 999, October 1978.

2. S. H. Crandall, W. D. Mark, Random Vibration in Mechanical Systems, Academic Press, 1963.

3. S. H. Crandall, W. Q. Zhu, Random Vibration: A Survey of Recent Developments, Journal of Applied Mechanics, Vol. 50, pp. 953 - 962, December 1983.

4. Y. K. Lin, Probablistic Theory of Structural Dynamics, McGraw-Hill, 1967.

5. A. Papoulis, Probability, Random Variables and Stochastic Processes, McGraw-Hill, 1965.

6. J. D. Robson, Results Arising Through the Matrix Formulation of Random Vibration Analysis, International Journal of Vehicle Design, Vol. 3, No. 3, pp. 297 - 306, 1982.

7. H. S. Ward, Dynamic Characteristics of a Multistory Concrete Building, The Institute of Civil Engineering, Vol. 43, pp. 553 - 572, August 1969.

APPENDIX A - TYPICAL PLANT VIBRATION DATA

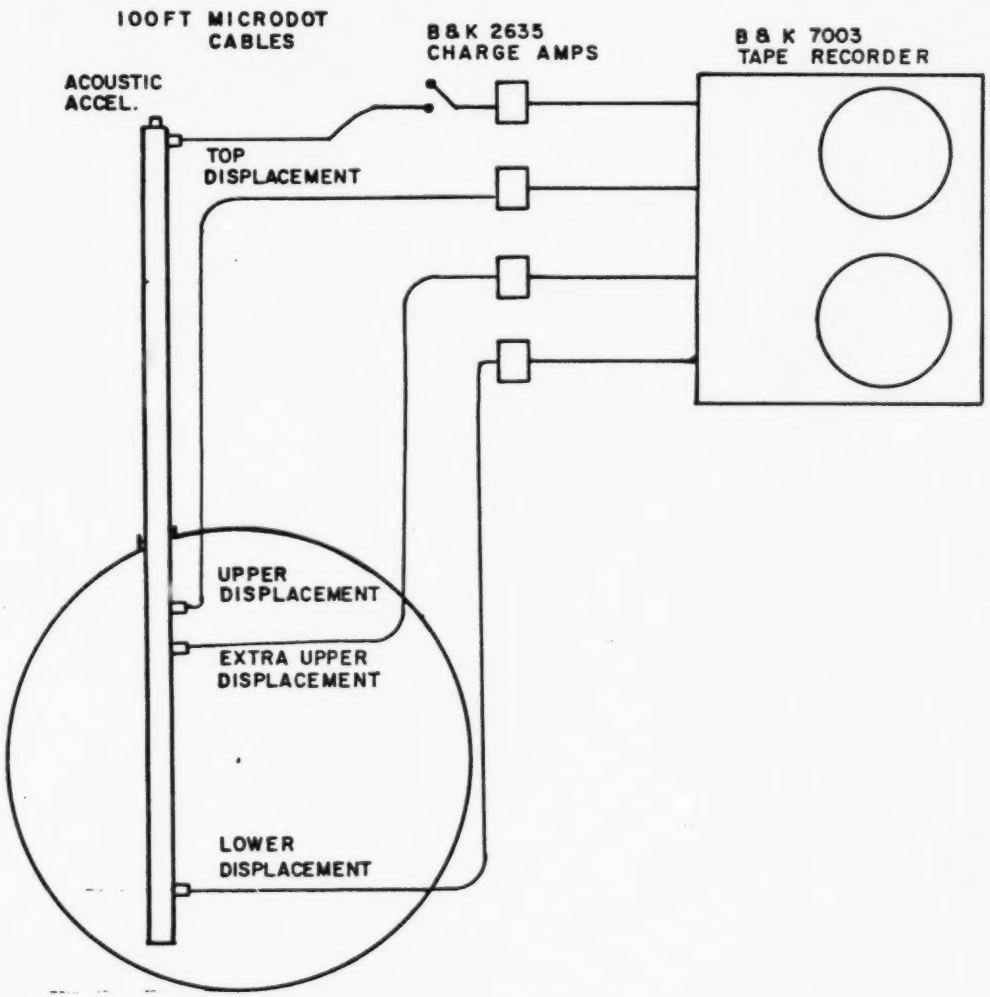


FIG. 19

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

U5 LZCU VIBRATION MONITORING

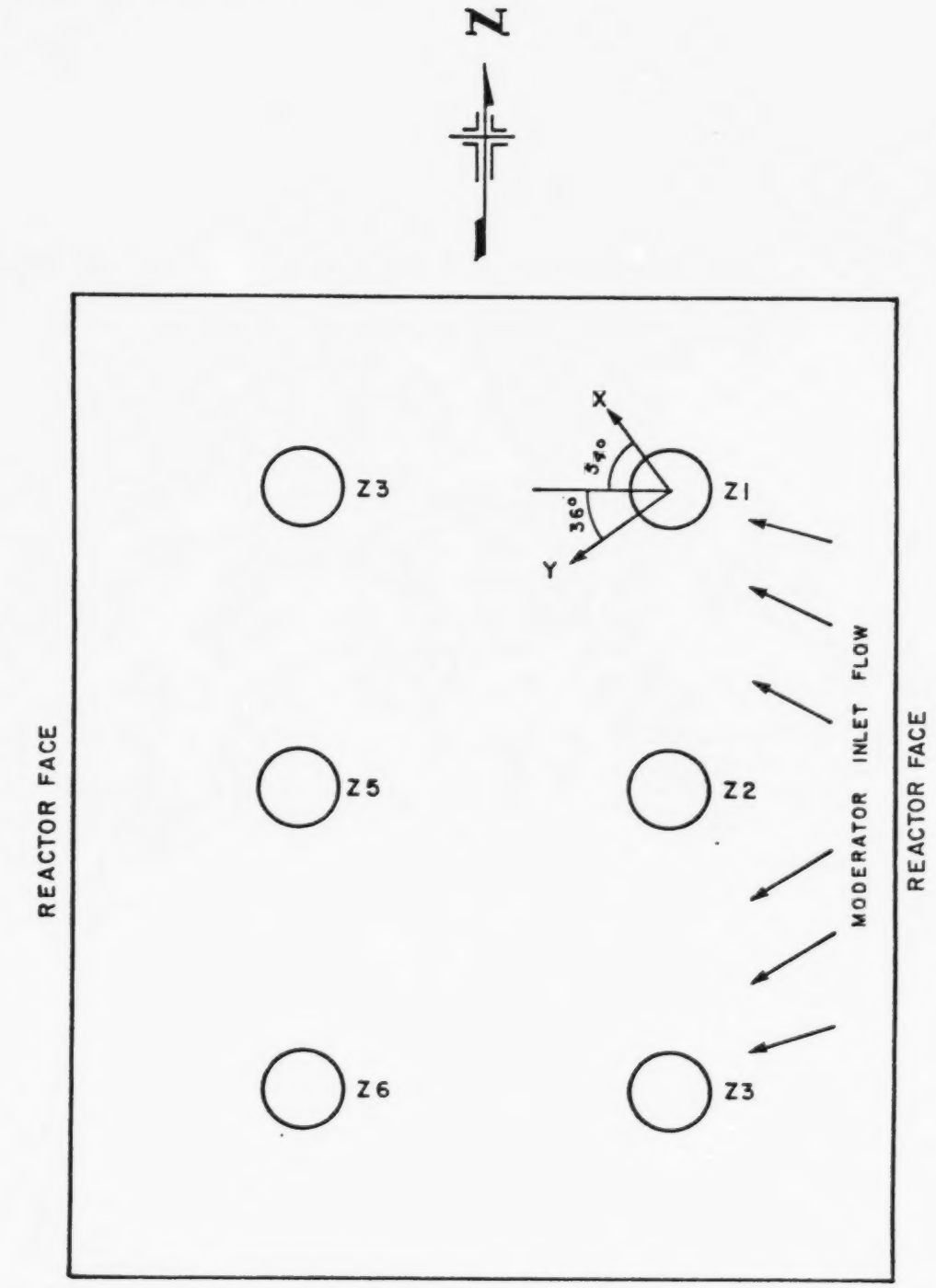


FIG. 20

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

ORIENTATION OF MONITORING EQUIPMENT FOR LZCU ZI UI

ACRES

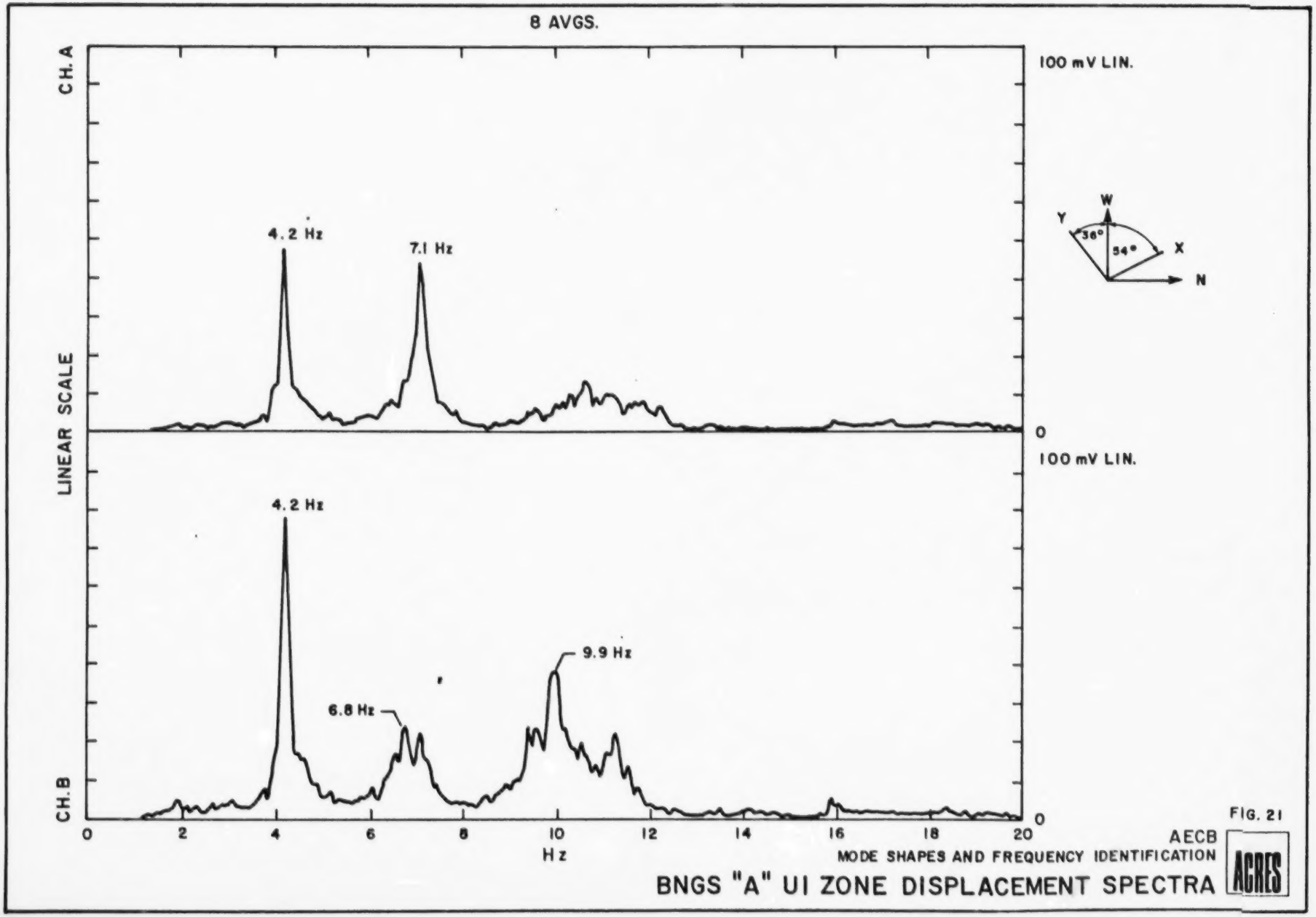


FIG. 21

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

BNGS "A" UI ZONE DISPLACEMENT SPECTRA

ACRES

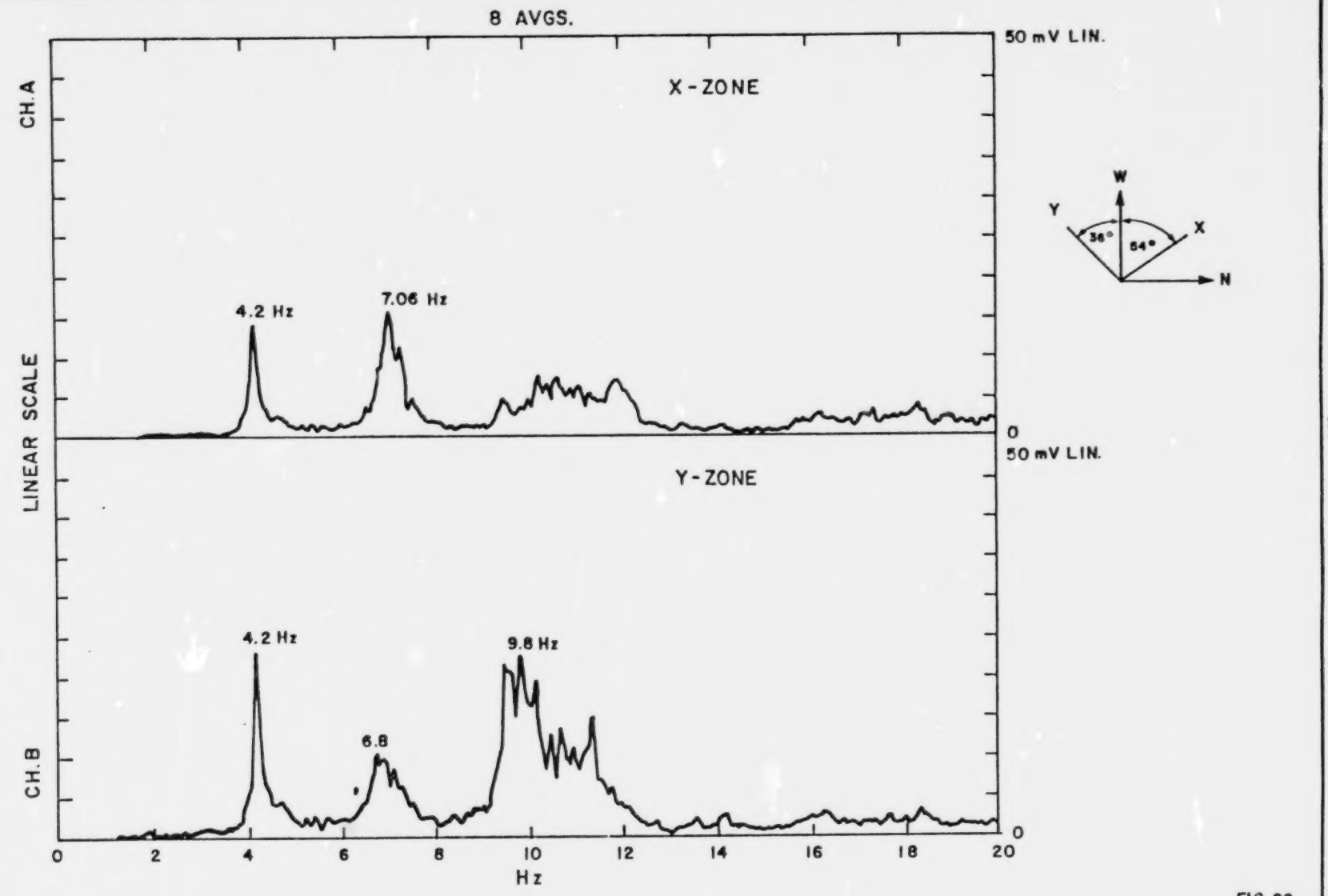


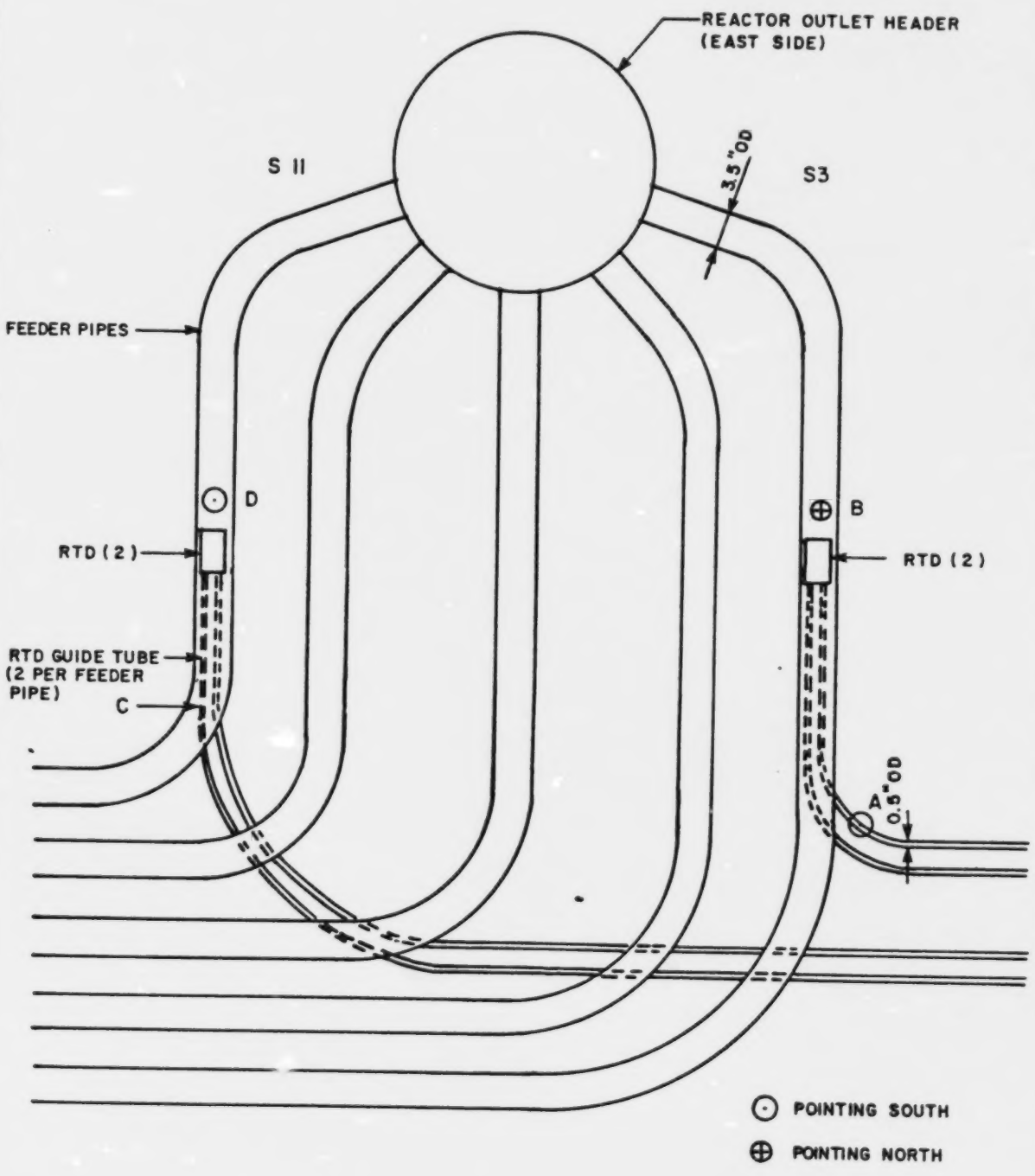
FIG. 22

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

BNGS "A" UI ZONE VELOCITY SPECTRA

ACRES



END VIEW LOOKING NORTH

FIG. 23

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

PICKERING GS - FEEDER PIPE AND GUIDE TUBE VIBRATION

ACRES

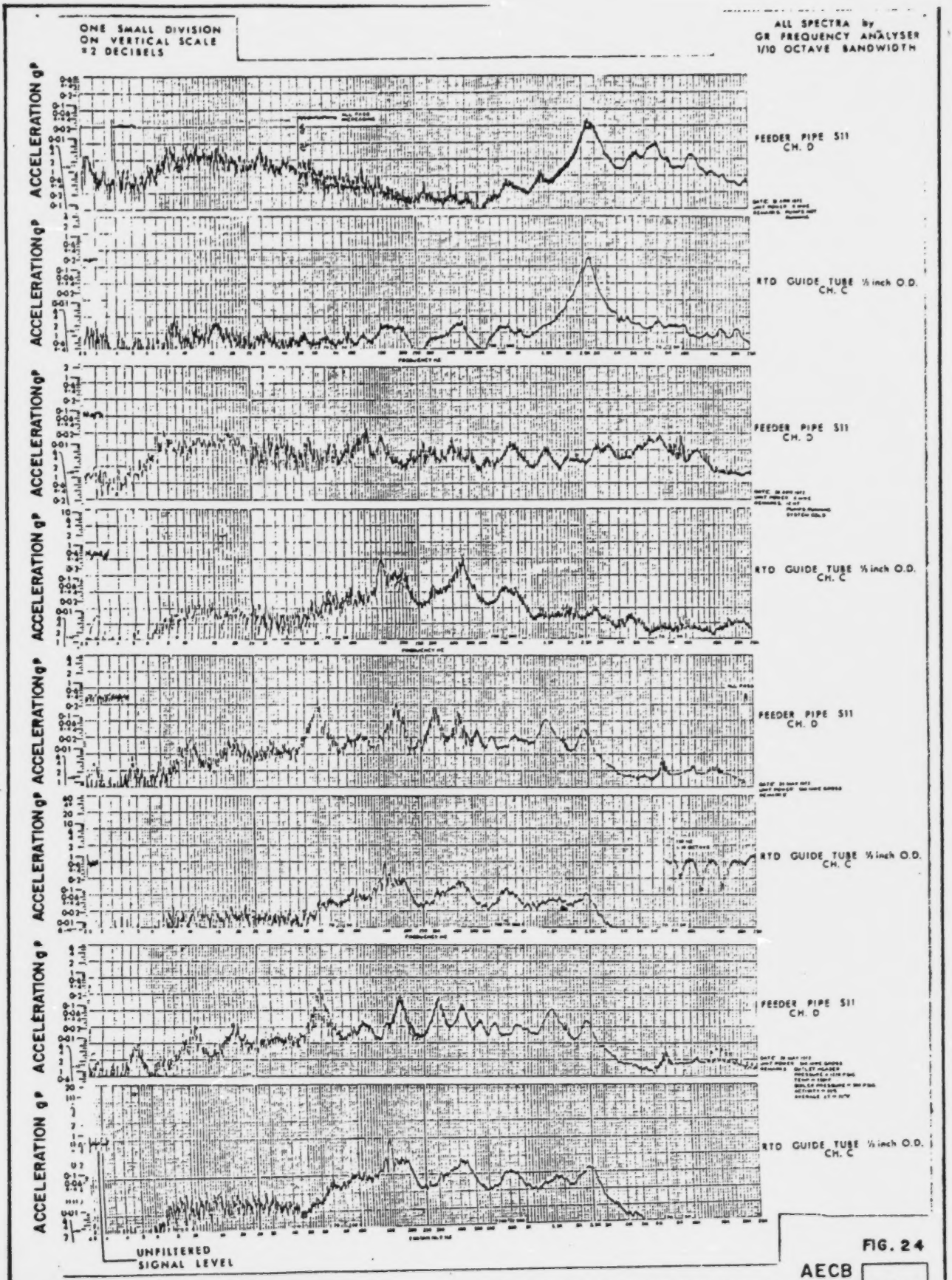

ONE SMALL DIVISION ON VERTICAL SCALE = 2 DECIBELS
ALL SPECTRA by GR FREQUENCY ANALYSER 1/10 OCTAVE BANDWIDTH
ACCELERATION g²
FEEDER PIPE S11 CH. D
RTD GUIDE TUBE ½ inch O.D. CH. C
UNFILTERED SIGNAL LEVEL

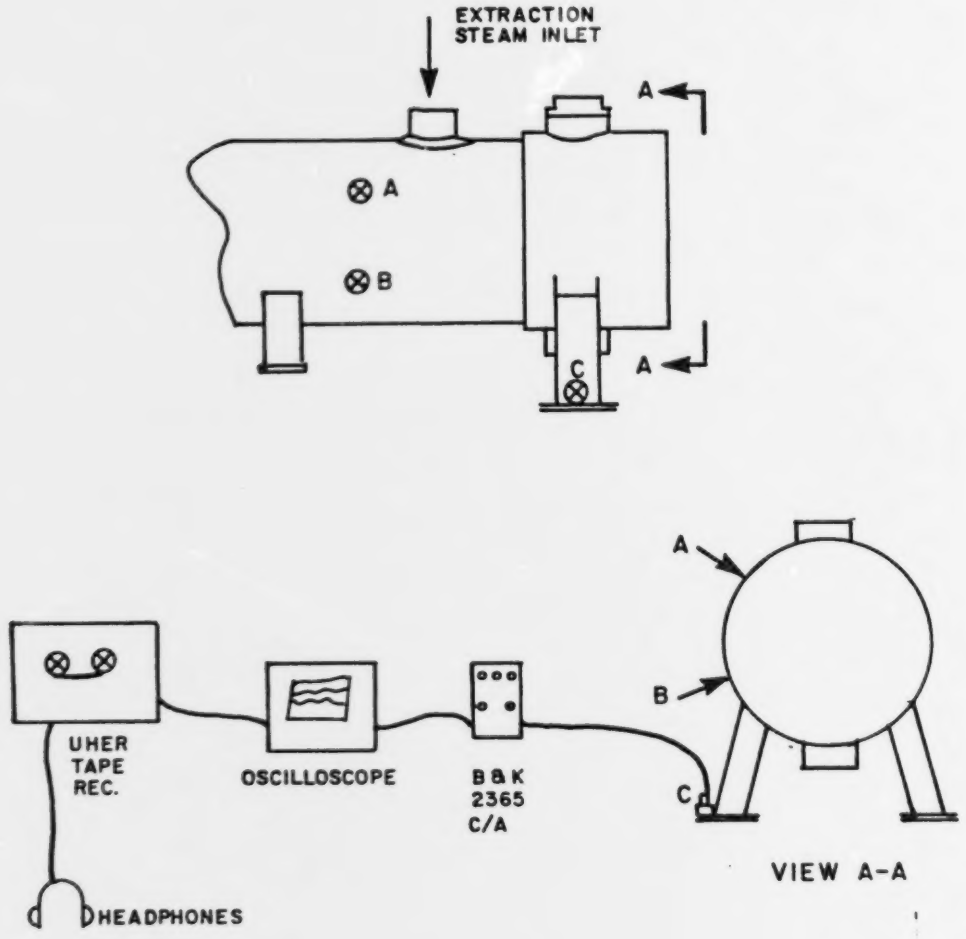
FIG. 24

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

PICKERING G.S. - FEEDER PIPE VIBRATION INVESTIGATION REACTOR No.3 EAST OUTET HEADER PIPES

AGRES



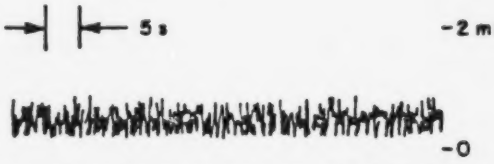
ACCELEROMETERS: A&B: TYPE 4368
4.56 pC/m/s²
C : TYPE 4370
10.03 pC/m/s²

FIG. 25

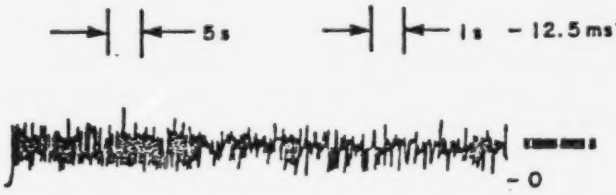
THUNDER BAY GS
UNIT 2 VIBROACOUSTIC MONITORING OF FEEDWATER HEATER No. 5

AECB
MODE SHAPES AND FREQUENCY IDENTIFICATION

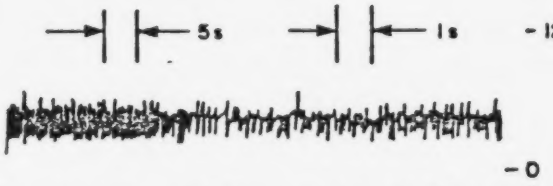
ACRES



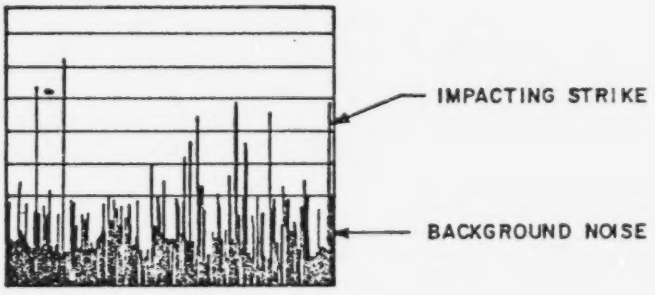
a) FULL-WAVE RECTIFIED VELOCITY SIGNAL AT POINT C



2a) FULL-WAVE RECTIFIED ACCELEROMETER SIGNAL AT POSITION "A" ON SHELL



2b) FULL-WAVE RECTIFIED ACCELEROMETER SIGNAL AT POSITION "B" ON SHELL



2c) PNGS B SHUTDOWN COOLER TUBE IMPACTING AND BACKGROUND NOISE

FIG. 26

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

MEASURED VIBRATIONS OF FEEDWATER HEATER No. 5 THUNDER BAY G.S. UNIT 2

ACRES

APPENDIX B - BACKGROUND THEORY

BACKGROUND THEORY

1. RESPONSE OF THE CONSTANT PARAMETER LINEAR SYSTEM TO RANDOM EXCITATIONS

A system has constant parameters when all properties of the system are invariant with respect to time. In the case of a structure it can be described by a system of second order ordinary differential equations.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \qquad (B.1.1)$$

The mass matrix [M], damping matrix [C], and the stiffness matrix [K] are constant. The force $\{F\}$ may depend on time, $\{F\} = \{F(t)\}$.

For further analysis of the system it is convenient to convert the set of n second order differential equations (B.1.1) to an equivalent set of 2n first order differential equations.

The new variables (referred to as state variables) are assigned to each original variable and their derivatives. The relations between state variables $y_i (i = 1, 2, \ldots, 2n)$ and original variables $x_i (i = 1, 2, \ldots, n)$ are the following

$$y_i = \dot{x}_i$$

$$y_{i+n} = x_i$$

In the vector notation, the same relations are

$$\begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{Bmatrix} \qquad \begin{Bmatrix} y_{n+1} \\ y_{n+2} \\ \vdots \\ y_{2n} \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$$

Equation B.1.1 can be replaced by a first order matrix equation for $y = [y_1(t), y_2(t),\ldots,y_{2n}(t)]$,

$$[A]\{\dot{y}\} + [B]\{y\} = \{Q\} \qquad (B.1.2)$$

where matrices [A], [B], and $\{Q\}$ are expressed in terms of mass matrix [M], damping matrix [C] and stiffness matrix [K]

$$[A] = \begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix}, \quad [B] = \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix},$$

$$\{Q\} = \begin{Bmatrix} \{0\} \\ \{F\} \end{Bmatrix}.$$

Equation B.1.2 is readily transformed into

$$\{\dot{y}\} + [G]\{y\} = \{P\} \qquad (B.1.3)$$

where

$[G] = [A]^{-1}[B]$ and $\{P\} = [A]^{-1}\{Q\}$

The response y of the system subjected to exciting forces is

$$y(t) = e^{-Gt}y(0) + \int_0^t e^{-G(t-\tau)} P(\tau)\, d\tau$$

If the initial conditions for y(t) are zeros $y(0) = 0$ or if the time t is large then the solution is

$$y(t) = \int_0^t e^{-G(t-\tau)}P(\tau)\, d\tau$$

The formal solution can be replaced by a form better suited for analysis.

$$y(t) = \int_0^t h(t-\tau)\, P(\tau)\, d\tau$$ Duhamel integral

where h(t) is an impulse response function of a system. Applying Fourier transform to the above formula we can obtain the response of the system expressed in terms of frequency.

$$\{Y(w)\} = [H(w)]\{P(w)\}$$

where [H(w)] is the 2n x 2n matrix of the frequency response functions.

When the linear system is subjected to random excitations, it is necessary to determine the random characteristics of tne input, i.e., its autocorrelation function R_{pp} and corresponding spectral density function. The autocorrelation function of a random ergotic process P(t) can be obtained directly from convolution integral.

$$R_{PP}(\tau) = \int_0^\infty P(t+\tau)\, P(\tau) d\tau$$

To obtain the random characteristic of the response of the system (Equation B.1.1) to random input $Q(\tau)$ consider the integral:

$$P(t)y(t+\tau) = \int_0^\infty h(\xi)P(t)P(t+\tau-\xi)\, d\xi$$

The expected value of the left side with respect to τ gives the cross-correlation function.

$$R_{Py}(\tau) = \int_0^\infty P(t)\, y(t+\tau)\, dt$$

The expected value of the right side of the integral yields relation

$$R_{Py}(\tau) = \int_0^\infty h(\xi)\, R_{PP}(\tau-\xi)\, d\xi$$

The Fourier transform of the above equation yields the power spectrum and cross-spectral relations for the linear system

$$\begin{aligned} S_{Py}(w) &= H(w)\, S_{PP}(w) \\ S_{yy}(w) &= |H(w)|^2\, S_{PP}(w). \end{aligned} \qquad \text{(B.1.4)}$$

If the cross-spectral density function $S_{Py}(w)$ and the autocorrelation function of the input can be measured then it is possible to identify the system from the relationship

$$H(w) = S_{Py}(w)/S_{PP}(w)$$

which contains information about the gain factor and the phase angle.

The results presented above comprise a most general approach to the analysis of the response of the linear system to stationary random excitations. In practice, very often it is assumed that the damping matrix [C] is a linear combination of mass matrix [M] and stiffness matrix [K]. This assumption substantially reduces the computational burden and it is implemented in all FEM structural dynamics programs.

2. DAMPING IN STRUCTURAL SYSTEMS

The mass matrix [M] and the stiffness matrix [K] are taken as symmetrical. A more detailed and accurate analysis requires the inclusion of damping matrix [C]. In the case of structural damping it is assumed that the [C] is a linear combination of matrices [M] and [K].

$[C] = \alpha[M]+\beta[K]$ $\quad \alpha,\ \beta$ = constants

Then the matrix [C] is also symmetrical.

For hysteresis damping it is assumed that the damping force is proportional to the elastic force, but as energy is dissipated, the force is in phase with velocity. Thus the damping matrix for a simple harmonic motion is given by

$$[C] = j\gamma[K]$$

where

$j = \sqrt{-1}$ and γ is a proportionality constant.

In this case the damping characteristics of the system are introduced into the analysis by modification of the stiffness matrix K which now becomes a complex $[K^1] = (1+j\gamma)[K]$. The modified Equation B.1.1 with hysteretic damping has a form

$$[M]\{\ddot{x}\} + (1+j\gamma)[K] = \{F\} \qquad (B.2.1)$$

To obtain an equivalent formulation of Equation B.2.1 in terms of state variables, the matrix [C] has to be taken as a null-matrix and the stiffness matrix [K] has to be replaced by modified complex stiffness matrix $[K^1] = (j\gamma)[K]$.

3. MODE SHAPES AND NATURAL FREQUENCIES OF THE STRUCTURAL LINEAR SYSTEM

The motion of a n degree of freedom system is described by a set of n ordinary differential equations of the second order

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\} + [K]\{x\} = \{F\} \qquad (B.3.1)$$

in which $\{\ddot{x}\}$, $\{\dot{x}\}$, and $\{x\}$ are vectors of acceleration, velocity, and displacement respectively.

$$\{\ddot{x}\} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_n \end{Bmatrix} \qquad \{\dot{x}\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{Bmatrix} \qquad \{x\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} .$$

Mass matrix [M] is symmetrical and so is the stiffness matrix [K]. Damping matrix [C] is also symmetrical by the fact that in common practice of structural dynamics [C] is taken as a linear combination of [M] and [K]. In the case of hysteretic damping the stiffness matrix is modified $[K] = (1+j\gamma)[K]$, and both matrices [M] and $[K^1]$ are symmetric. The variables $\{x\}$ are coupled by the fact that either the mass matrix [M] or stiffness matrix [K] (or $[K^1]$) or both of them are nondiagonal.

Nondiagonal matrix [M] indicates that there is a dynamic coupling between variables whereas nondiagonal stiffness matrix [K] indicates static (or elastic) coupling.

Decoupling the system of Equation B.3.1 can be done by introducing a linear transformation $[\phi]$ of the current variables $\{x\}$ such that the new variable $\{Z\}$ are uncoupled. The desired transformation is

$$\{x\} = [\phi]\{Z\} \qquad (B.3.2)$$

To determine the transformation $[\phi]$ the mode shapes and the natural frequences of the system (Equation B.3.1) have to be found. In our system the type of damping is either structural or hysteresis so the mode shapes of damped system are identical to those of undamped system. The eigenvalues λ of the system (Equation B.3.1) can be obtained by solving the characteristic equation of an undamped system

$$\det\left|[M]^{-1}[K] - \lambda[I]\right| = 0$$

Substituting each eigenvalue λ_i ($i = 1,2,\ldots,n$) into the equation

$$[[M]^{-1}[K] - \lambda_i[I]]\{u\}_i = 0$$

the set of n eigenvectors (mode shapes) can be determined.

Finally, the desired linear transformation has a form

$$[\phi] = \left[\begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix}_1 \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix}_2 \cdots \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix}_n \right] = [u_{ij}]$$

The eigenvectors are orthogonal and thus the mass matrix [M] and stiffness matrix [K] can be diagonalized.

$$[\phi]^T [M] [\phi] = [M]$$

$$[\phi]^T [K] [\phi] = [K]$$

Applying transformation $[\phi]$ to the damping matrix [C] yields diagonal matrix [C].

$$[\phi]^T [C] [\phi] = \alpha [M] + \beta [K] = [C]$$

Finaily the system of Equation B.3.1 can be replaced by set of n uncoupled differential equations of the second order

$$[M] \{\ddot{Z}\} + [C] \{\dot{Z}\} + [K] \{Z\}^T = [\phi] \{F\} \quad \text{(B.3.3)}$$

or in equivalent form

$$\ddot{Z}_i + 2\xi_i w_i \dot{Z}_i + w_i^2 Z_i = P_i/M_i \quad \text{(B.3.4)}$$

where

$$w_i^2 = K_i/M_i$$

$$\xi_i = \frac{C_i}{2\sqrt{K_i M_i}} \quad , \quad P_i = \sum_{j=1}^{n} u_{ji} F_i$$

The new variables Z_i are called principal coordinates or modal coordinates. According to Equation B.3.2 the original variables x_i can be expressed in terms of modal coordinates

$$x_i(t) = \sum_{j=i}^{n} u_{ij} Z_j(t) \qquad (B.3.5)$$

Our interest is to determine the relationship between exciting force vector $\{F(t)\}$ and the response of the system $\{x(t)\}$ in the time domain and the frequency domain $\{x(w)\}$.

Taking the Fourier transform of both sides of Equation B.3.4, the frequency response of a single principal mode to excitation force F can be obtained.

$$[-w^2 + j\,2\,\xi_i w_i w + w_i^2]\, Z_i(w) = P_i(w)/M_i$$

$$Z_i(w) = \frac{P_i(w)}{M_i(-w^2 + j2\xi_i w_i w + w_i^2)}$$

If the exciting force is a unit impulse function $P_i(t) = \delta(t)$, then its Fourier transformer is equal to 1 and the $Z_i(w)$ represents the frequency response function of the mode i

$$h_i(w) = \frac{1}{M_i\,(-w^2 + j2\xi_i w_i w + w_i^2)} \qquad (B.3.6)$$

where

$M_i = 1$ for normalized eigenvectors.

The equivalent form of the frequency response function in time domain is

$$h_i(t) = \frac{1}{w_i\sqrt{1-\xi_i^2}} \exp(-\xi_i w_i t) \sin(w_i \sqrt{1-\xi_i^2}\, t) \qquad (B.3.7)$$

The new variables Z_i are called principal coordinates or modal coordinates. According to Equation B.3.2 the original variables x_i can be expressed in terms of modal coordinates

$$x_i(t) = \sum_{j=i}^{n} u_{ij} Z_j(t) \qquad \text{(B.3.5)}$$

Our interest is to determine the relationship between exciting force vector $\{F(t)\}$ and the response of the system $\{x(t)\}$ in the time domain and the frequency domain $\{x(w)\}$.

Taking the Fourier transform of both sides of Equation B.3.4, the frequency response of a single principal mode to excitation force F can be obtained.

$$[-w^2 + j\,2\,\xi_i w_i w + w_i^2]\, Z_i(w) = P_i(w)/M_i$$

$$Z_i(w) = \frac{P_i(w)}{M_i(-w^2 + j2\xi_i w_i w + w_i^2)}$$

If the exciting force is a unit impulse function $P_i(t) = \delta(t)$, then its Fourier transformer is equal to 1 and the $Z_i(w)$ represents the frequency response function of the mode i

$$h_i(w) = \frac{1}{M_i\,(-w^2 + j2\xi_i w_i w + w_i^2)} \qquad \text{(B.3.6)}$$

where

$M_i = 1$ for normalized eigenvectors.

The equivalent form of the frequency response function in time domain is

$$h_i(t) = \frac{1}{w_i\sqrt{1-\xi_i^2}} \exp(-\xi_i w_i t) \sin(w_i \sqrt{1 - \xi_i^2}\, t) \qquad \text{(B.3.7)}$$

$\{F(w)\}$ = is a vector of exciting forces.

Rearranging Equation B.3.9 a more convenient form can be obtained.

$$x_i(w) = \sum_{j=1}^{n} H_{ij}(w)\, F_j(w)$$

$$\{x(w)\} = [H(w)]\,\{F(w)\} \qquad (B.3.11)$$

where

$$H_{ij}(w) = \sum_{k=1}^{n} u_{ik}\, h_k(w)\, u_{jk}$$

is a frequency response function of the DOF i to the excitation force j.

For random excitations when the matrix spectral density function of exciting forces is$[S_{FF}(w)]$ the response can be obtained as a cross-spectral density of the input and output $[S_{Fx}(w)]$

$$[S_{Fx}(w)] = [H(w)][S_{FF}(w)] \qquad (B.3.12)$$

or as a spectral density function of the output

$$[S_{xx}(w)] = |H(w)|^2\, [S_{FF}(w)] \qquad (B.3.13)$$

At each particular frequency w_1 the frequency response function (FRF) can be obtained from Equation B.3.12.

$$H(w_1) = [S_{Fx}(w_1)][S_{FF}(w)_1]^{-1} \qquad (B.3.14)$$

Much useful information about the system can be obtained by analyzing the spectral density matrices $[S_{xx}]$ and $[S_{FF}]$. If off-diagonal elements of the matrix $[S_{FF}]$ are zeros, it means that there is no correlation between exciting forces which is most often the case. On the other hand, if the off-diagonal elements of the matrix $[S_{xx}]$ are zeros

it means that there is no correlation between DOFs and each node can be seen as an independent DOF.

In some cases, it is useful to assess the level of correlation between DOFs and the exciting forces.

The quantitative estimation of an internal correlation between DOFs and the exciting forces gives the coherence function

$$\gamma^2_{Fx}(w) = \frac{\left|S_{Fx}(w)\right|^2}{S_{FF}(w)\, S_{xx}(w)} \qquad (B.3.15)$$

The value of γ^2 is always between 0 and 1. For γ^2 near 1.0 there is a strong correlation between x and f. If F(t) and x(t) are completely unrelated, the coherence function will be zero. If the coherence function is greater then zero but less than unity, one or more of three possible situations exist.

- Extraneous noise is present in the measurements.
- The system relating F(t) and x(t) is not linear.
- x(t) is an output due to an input F(t) as well as to other inputs.

Usually the coherence function is calculated to identify internal coupling in the system.

4. ESTIMATION OF FREQUENCY RESPONSE FUNCTION

For the measured vectors of exciting forces $\{F\}$ and the corresponding response displacements vector $\{x\}$, the best estimate of the FRF $[H]$ is the one that minimizes the error vector $\{\varepsilon\}$ in the equation

$$\{x\} = [H]\{F\} + \{\varepsilon\} \qquad (B.4.1)$$

There are three accepted estimates of FRF H. The H_1 estimator is derived from the condition that the error vector $\{\varepsilon\}$ should be incoherent with the input force vector $\{F\}$. This condition written in terms of spectra is

$$S_{\xi F}(w) = 0 \tag{B.4.2}$$

The estimator of FRF H_1 which satisfies this condition is [13,14]

$$[H_1(w)] = [S_{xF}(w)]/[S_{FF}(w)] \tag{B.4.3}$$

The second estimator H_2 can be derived from conditions that the error vector $\{\varepsilon\}$ should be incoherent with the response vector $\{x\}$. It gives the defining equation for H_2

$$S_{xx}(w) = H(w)\, S_{Fx}(w) \tag{B.4.4}$$

The unique solution for H(w) can be thus obtained only if $[S_{Fx}(w)]^{-1}$ exists. It requires $[S_{Fx}(w)]$ to be square matrix, meaning that the number of force channels must equal the number of response channels. This requirement is too rigid, since most often the number of exciting forces is less than number of degree of freedom. Estimator H_2 however, is useful in single input-output system identification.

Recently the method that employs the tensorial approach for estimation of H(w) has been developed [Ref. 14].

The basic idea behind the H_v estimator is to minimize the error vector in a coordinate independent way. It can be achieved by minimizing the trace of the spectral matrix $S_{\varepsilon\varepsilon}(w)$ of the error vector $\{\varepsilon\}$. Since $S_{\varepsilon\varepsilon} = E(\varepsilon\varepsilon^t)$, the spectral matrix of the error vector is determined.

$$S_{\varepsilon\varepsilon} = S_{xx} + HS_{FF}H^T - HS_{FF} - S_{xF}H^T \tag{B.4.5}$$

Since the trace of the matrix $S_{\varepsilon\varepsilon}$ is needed the diagonal elements are

$$S^{ii}_{\varepsilon\varepsilon} = S^{ii}_{xx} + \sum_{k=1}^{n} H^2_{ik}\, S^{k}_{FF} - 2 \sum_{k=1}^{n} H_{ik}\, \mathrm{Re}[S^{ki}_{xF}] \tag{B.4.6}$$

where

$S^{ii}_{\varepsilon\varepsilon}$ = diagonal elements of the matrix $S_{\varepsilon\varepsilon}$

S^{ii}_{xx} = diagonal elements of output spectral density matrix S_{xx}

H_{ik} = elements of the FRF

$Re[S^{ki}_{xF}]$ = real part of cross-spectral density matrix S_{xF}.

The estimate of H(w) is the one that minimizes the trace of the matrix $S_{\varepsilon\varepsilon}$

$$tr\ (S_{\varepsilon\varepsilon}) = \sum_{i=1}^{n} S^{ii}_{\varepsilon\varepsilon}$$

Since the trace of the matrix is invariant under coordinate transformation, this is a tensorial criterion. If, for example, incoherent noise of equal amplitude is present in all channels, the noise spectrum is a diagonal matrix that only translates the eigenvectors of $S_{\varepsilon\varepsilon}$ without changing them. The H estimate is therefore unaffected by such noise. For single input-output system the H obtained from this criterion is between H_1 and H_2.

4.1 Confidence Interval for Frequency Response Function Estimate

The confidence interval for frequency response function estimate H(w) [Ref. 13] with confidence coefficient $1-\alpha$ can be determined by a quantity r(w) such that

$$\left|H(w) - H(w)\right| < r^2(w) \qquad (B.4.1.1)$$

$$r^2(w) = \frac{2}{n-2} F_{2,n-2,\alpha} [1-\gamma^2_{FF}(w)] \frac{S_{xx}(w)}{S_{FF}(w)} \qquad (B.4.1.2)$$

where

$n=2B_eT$ = number of degrees of freedom of each spectral estimate

B_e = frequency bandwidth resolution (Hz)

T = record length (s)

$F_{2,n-2,\alpha}$ = 100α percentage point of an F distribution with $n_1 = 2$ and $n_2=n-2$ degrees of freedom.

$S_{xx}(w)$ = power spectrum estimate of the output.

$S_{FF}(w)$ = power spectrum estimate of the input.

$\gamma^2_{Fx}(w)$ = sample estimate of the ordinary coherence function between the input F(t) and output x(t).

The approximate $1-\alpha$ confidence intervals for the amplitude and phase of FRF at each frequency w are given by

$H(w) = H(w) \pm r(w)$ - amplitude

$\psi(w) = \psi(w) \pm \Delta\psi(w)$ - phase

where

$\Delta\psi(w) = \sin^{-1}[r(w)/H(w)]$

5. EXTRACTION OF MODAL PARAMETERS USING DYNAMIC STIFFNESS METHOD

The frequency response function of a pure mode is

$$H(w) = \frac{A}{w_1^2 - w^2 + 2\xi w_r w j} \qquad (B.5.1)$$

A = constant
w_r = resonant frequency
ξ = damping ratio
w = frequency
$j = \sqrt{-1}$

To determine the unknown modal parameters Equation B.5.1 has to be fitted to given measurements $H(w_1)$, $H(w_2),\ldots,H(w_n)$ of the FRF. The fitting algorithm can be significantly simplified if instead of Equation B.5.1 the dynamic stiffness if fitted.[Ref. 15] The dynamic stiffness of a pure mode

$$K(w) = 1/H(w) = \frac{w_r^2 - w^2 + 2\xi w w_r j}{A} = Re(w) + j.Im(w)$$

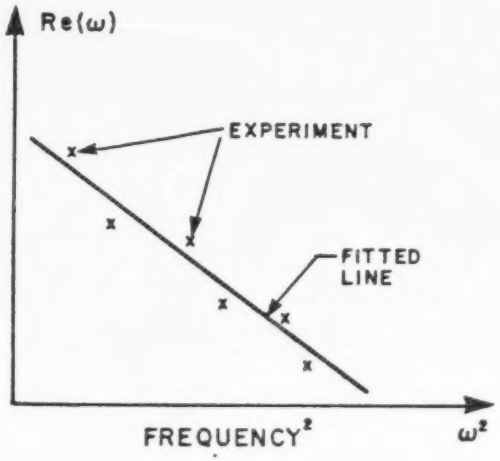
is a linear function of w^2 and w, thus the simple regression method (Figure 27) may be used to identify the modal paramters A, w_r and ξ. Results are exact for pure mode response, but if the data contains combined response of several modes the special mode separation technique is necessary.[Ref. 15] An example of frequency response function data was generated using $A=1$, $w_r=1.5$ Hz and $\xi=0.05$. Using the dynamic stiffness method these parameters were identified as shown in Figure 28 and Figure 29. Obtained estimates of A, w_r and ξ are the same because only one mode was analysed.

6. KENNEDY-PANCU METHOD OF MODAL PARAMETERS IDENTIFICATION

For a single degree of freedom system the frequency response function [1]

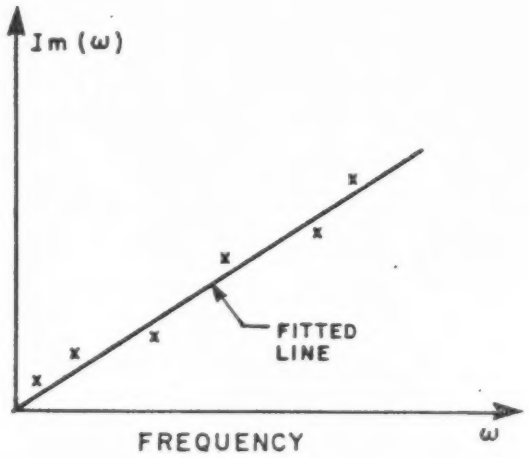
$$H = \frac{A}{w_r^2 - w^2 + 2\xi w_r w j} \qquad (B.6.1)$$

represents a circle when plotted on a Nyquist diagram (Figure 30). The natural frequency w_r of the system is a frequency on the circle where the



$$\text{SLOPE} = -\frac{1}{A}$$

$$\text{INTERCEPT} = \frac{\omega_r^2}{A}$$



$$\text{SLOPE} = \frac{2\xi\omega_r}{A}$$

FIG. 27

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

DYNAMIC STIFFNESS METHOD

ACRES

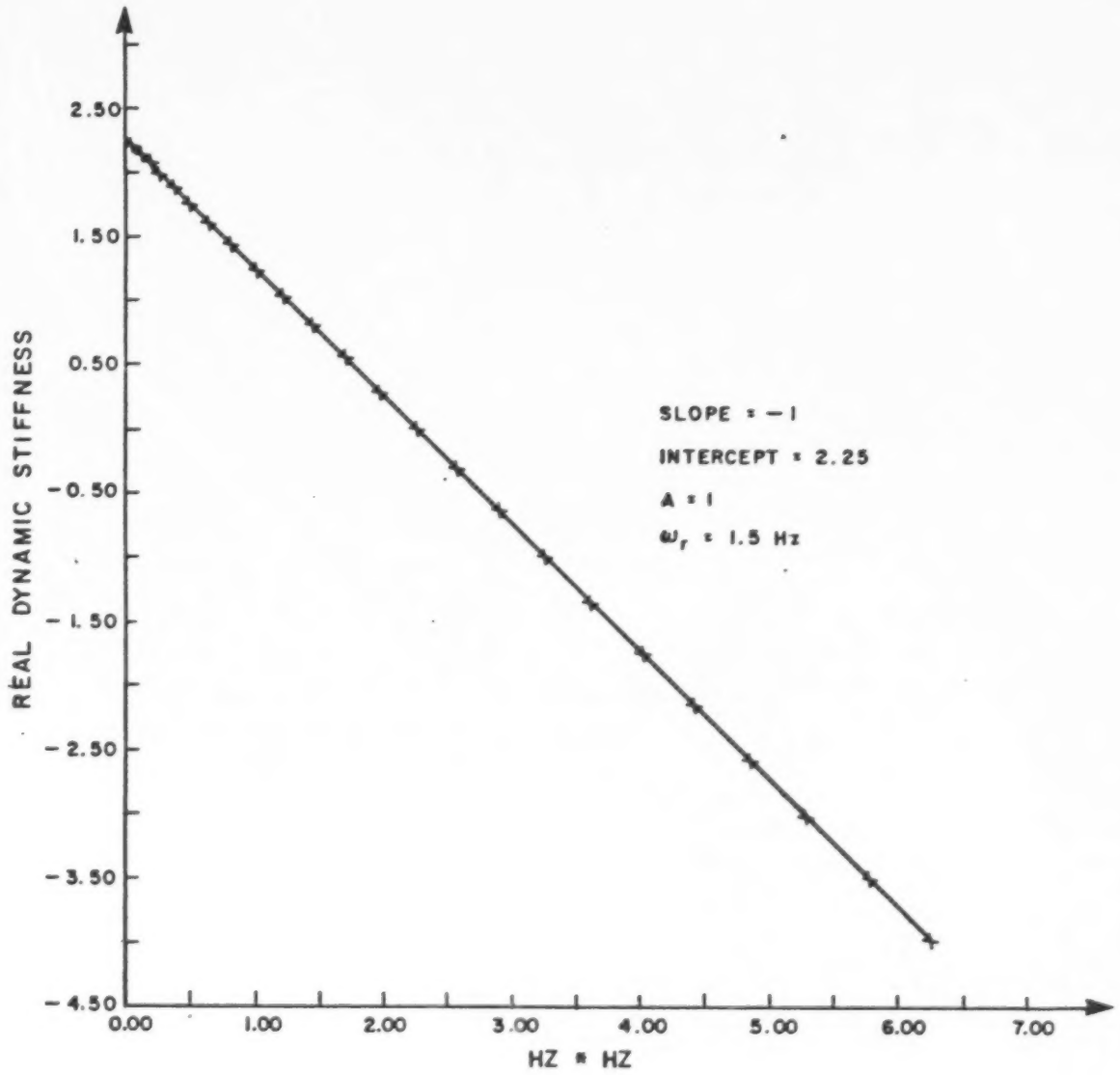


FIG. 28

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

IDENTIFICATION OF AMPLITUDE AND NATURAL FREQUENCY

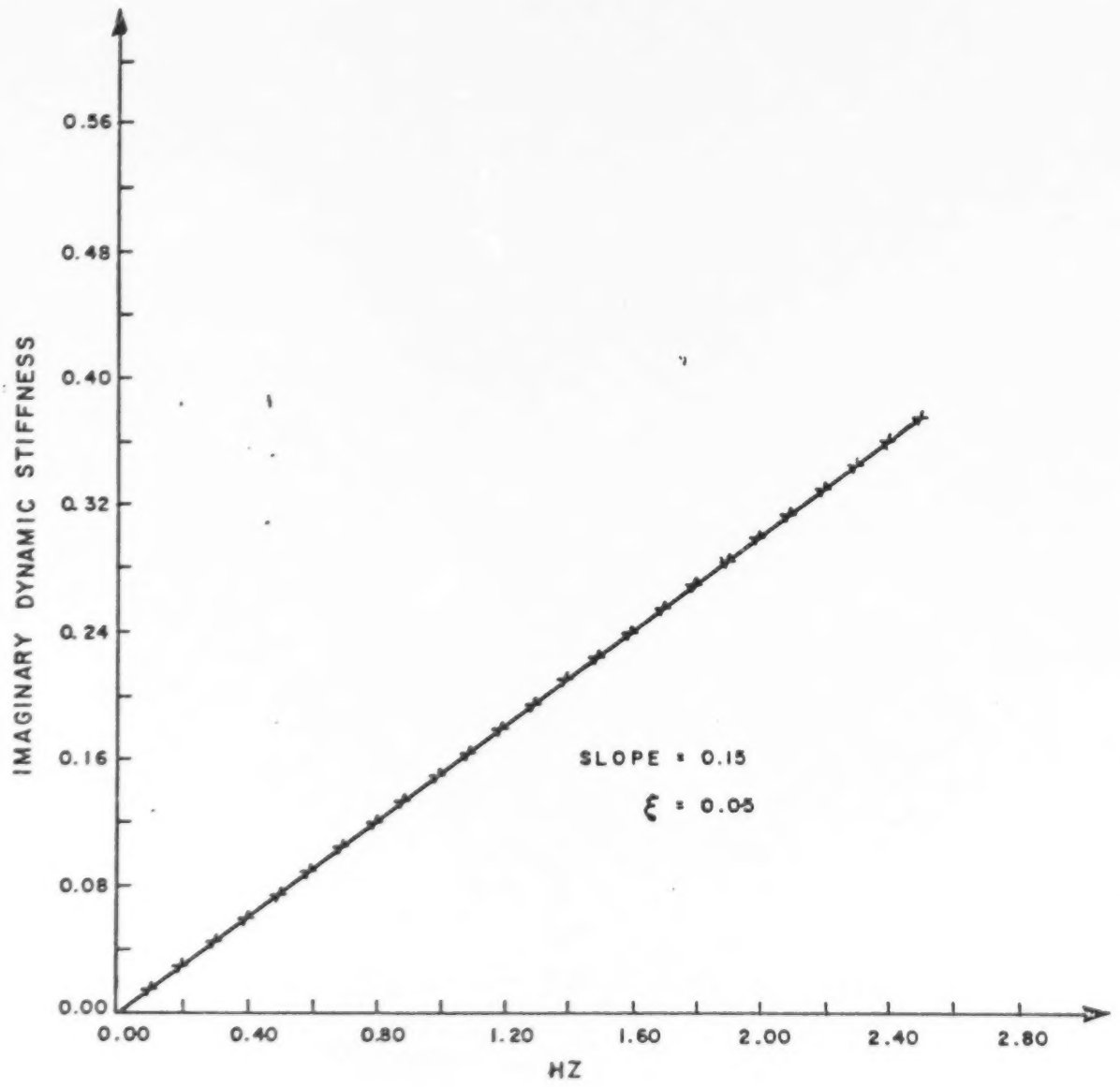


FIG. 29

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

IDENTIFICATION OF DAMPING RATIO

ACRES

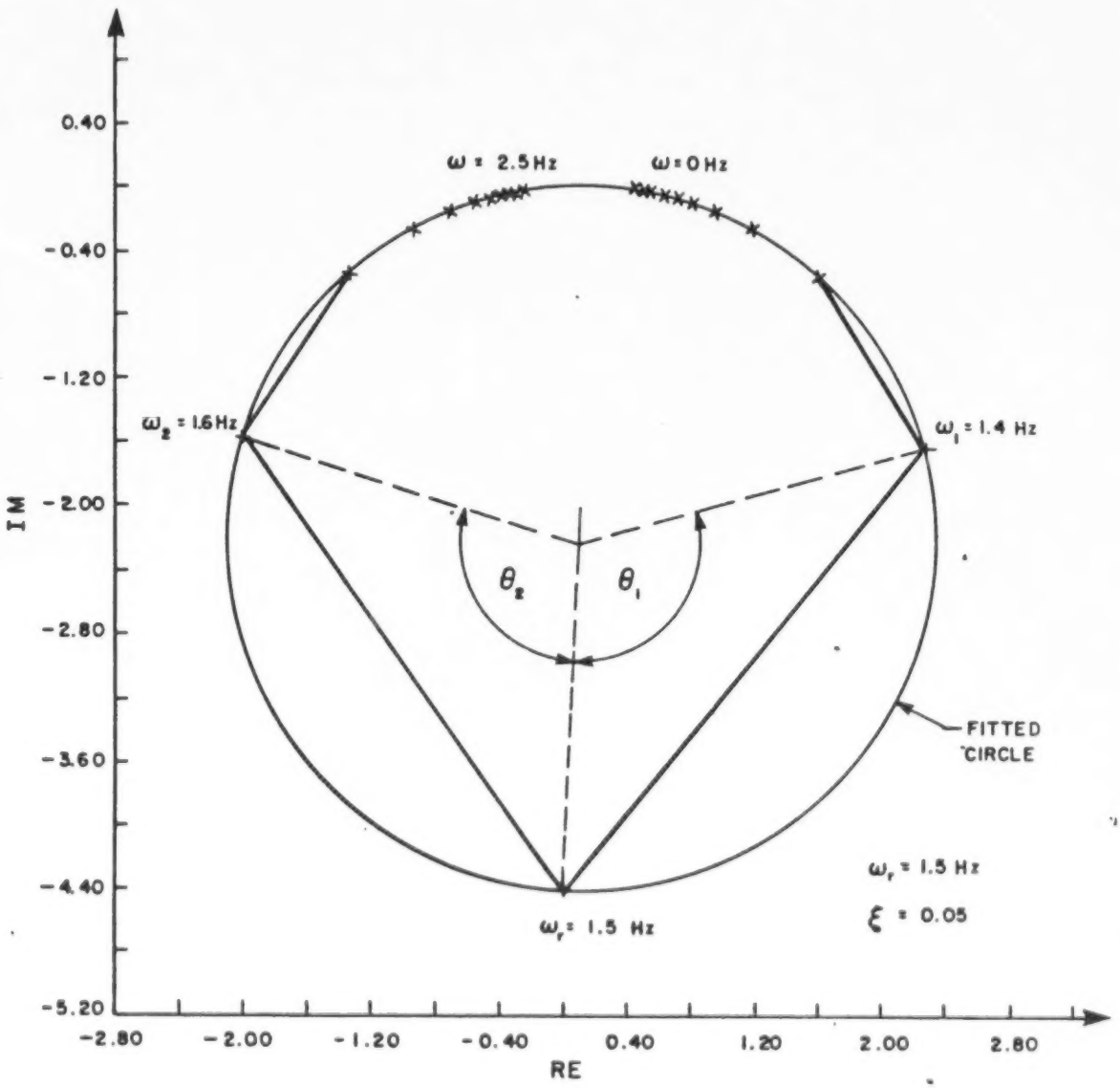


FIG. 30

AECB

MODE SHAPES AND FREQUENCY IDENTIFICATION

NYQUIST PLOT-OF A PURE MODE

ACRES

rate of change of arc length is maximum. The damping ratio ξ and the constant A are determined from relations.[Ref. 1, 15]

$$2\xi w_r = \frac{w_2^2 - w_1^2}{w_r^2} \left[\frac{1}{\tan 0.5\theta_1 + \tan 0.5\theta_2}\right] \qquad (B.6.2)$$

$A = 4w_r^3 \xi$ x radius of circle

The accuracy of results thus obtained are affected by noise in data. This effect increases as the frequency of excitation approaches the resonant frequency. Another disadvantage of the method is that the mode constant A is computed using ξ and w_r (Equation B.6.2). For lightly damped structures with large relative magnitude of w to ξ, significant errors may be present in modal parameter estimates.

APPENDIX C - COMPUTER PROGRAMS

```
C
C     MODE SHAPE AND FREQUENCY IDENTIFICATION
C         FOR SEISMIC ANALYSIS
C
C     DECEMBER 1985
C
C     ATOMIC ENERGY OF CANADA BOARD (FILE: R110Z4242Z4)
C     ACRES INTERNATIONAL LIMITED   (FILE: P744300)
C
C     FREQUENCY RESPONSE FUNCTION ESTIMATE
C     INPUT AND OUTPUT AUTOCORRELATION
C     INPUT AND OUTPUT SPECTRAL DENSITY
C     INPUT-OUTPUT CROSS-CORRELATION
C     COHERENCE FUNCTION ESTIMATE
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(7000),Y(7000),R(200),G(200)
     &,AR(200),BR(200),CR(200),QR(200),U(5)
      COMMON/AAA/PI
      PI=3.141592654
C
C     READ INPUT DATA
C     N     = NUMBER OF POINTS IN TIME SERIE
C     H     = TIME STEP (SEC)
C     L     = LAG TIME LENGHT
C     NODES = NUMBER OF NODES
C     NODE  = NODE NUMBER TO BE ANALYSED
C
      READ(5,*)N,H,L,NODES,NODE
C
C     FRQ = FREQUENCY INTERVAL
C
      FRQ=1.0/(H*(L-1))
C
C     READ EXCITING INPUT X(1...N)
C     AND STRUCTURE RESPONSE OF EACH NODE U(1...NODES)
C
      DO 10 I=1,N
      READ(5,*)X(I),(U(K),K=1,NODES)
      Y(I)=U(NODE)
   10 CONTINUE
C
C     COMPUTE AUTOCORRELATION OF INPUT X AND STORE IN R
C
      CALL AUTOCO(X,N,L,R)
C
C     COMPUTE EVEN (AR) AND ODD (BR) PARTS
C     OF CROSS-CORRELATION FUNCTION
C     AR(R)=(RXY(R*H)+RYX(R*H))/2
C     BR(R)=(RXY(R*H)-RYX(R*H))/2
C
      CALL CROSCO(X,Y,N,L,AR,BR)
C
C     COMPUTE SPECTRAL DENSITY G(F)
C     OF INPUT RXX(TAU)
C
      CALL AUTOSP(L,H,R,G)
C
C     COMPUTE COINCIDENT-SPECTRAL DENSITY CR(F)
C     AND QUADRATURE-SPECTRAL DENSITY QR(F)
C     BY FOURIER TRANSFORM OF AR(TAU) AND BR(TAU)
C
      CALL CROSSP(L,H,AR,BR,CR,QR)
```

```
C
C     PRINT INPUT X FOR PLOTTING
C
      CC=99.99
      WRITE(7,57)
      WRITE(7,53)
      WRITE(7,50)
      WRITE(7,56)
      WRITE(7,58)1,1
      WRITE(7,60)
      DO 20 I=1,500
      P=H*(I-1)
   20 WRITE(7,59) P,X(I)
      WRITE(7,59)CC,CC
C
C     PRINT STRUCTURE RESPONSE Y FOR PLOTTING
C
      WRITE(10,57)
      WRITE(10,53)
      WRITE(10,62)
      WRITE(10,56)
      WRITE(10,58)2,1
      WRITE(7,60)
      DO 30 I=1,500
      P=H*(I-1.)
   30 WRITE(10,59) P,Y(I)
      WRITE(10,59)CC,CC
C
C     PRINT AUTOCORRELATION OF INPUT
C     FOR PLOTTING
C
      WRITE(8,57)
      WRITE(8,54)
      WRITE(8,51)
      WRITE(8,56)
      WRITE(8,58)3,3
      WRITE(8,60)
      DO 21 I=1,L
      P=H*(I-1)
   21 WRITE(8,59)P,R(I)
      WRITE(8,59)CC,CC
C
C     PRINT EVEN (AR) AND ODD (BR) PARTS
C     OF THE CROSS-CORRELATION FUNCTION
C
      WRITE(8,60)
      DO 31 I=1,L
      P=H*(I-1.)
   31 WRITE(8,59) P,AR(I)
      WRITE(8,59)CC,CC
      WRITE(8,60)
      DO 32 I=1,L
      P=H*(I-1.)
   32 WRITE(8,59) P,BR(I)
      WRITE(8,59)CC,CC
C
C     PRINT SPECTRAL DENSITY G(F)
C     OF THE INPUT X(T)
C
      WRITE(9,57)
      WRITE(9,55)
      WRITE(9,52)
      WRITE(9,56)
```

```
      WRITE(9,58)4,3
      WRITE(9,60)
      DO 22 I=1,L
      P=(I-1)*FRQ/2.
   22 WRITE(9,59)P,G(I)
      WRITE(9,59)CC,CC
C
C     PRINT THE COINCIDENT-SPECTRAL DENSITY CR(F)
C     AND   THE QUADRATURE-SPECTRAL DENSITY QR(F)
C
      WRITE(9,60)
      DO 23 I=1,L
      P=(I-1.)*FRQ/2.
      Q=CR(I)
   23 WRITE(9,59) P,Q
      WRITE(9,59)CC,CC
      WRITE(9,60)
      DO 24 I=1,L
      P=(I-1.)*FRQ/2.
      Q=-QR(I)
   24 WRITE(9,59) P,Q
      WRITE(9,59)CC,CC
C
C     PRINT THE AMPLITUDE OF FREQUENCY RESPONSE FUNCTION
C     FRF(I)=SQRT(CR(I)**2+QR(I)**2)/G(I)
C
      WRITE(6,57)
      WRITE(6,55)
      WRITE(6,61)
      WRITE(6,56)
      WRITE(6,58)5,1
      WRITE(6,60)
      DO 25 I=1,L
      P=(I-1.)*FRQ/2.
      Q=SQRT(CR(I)**2+QR(I)**2)/G(I)
      WRITE(6,59) P,Q
   25 CONTINUE
      WRITE(6,59)CC,CC
C
C     PRINT REAL PART AND IMAGINARY PART
C     OF FREQUENCY RESPONSE FUNCTION FOR
C     NYQUIST PLOT OF FRF
C
      WRITE(11,67)L
      DO 26 I=1,L
      WW=(I-1)*FRQ/2.
      P=CR(I)/G(I)
      Q=-QR(I)/G(I)
      WRITE(11,59) WW,P,Q
   26 CONTINUE
C
C     PRINT THE PHASE OF FRF
C     PHASE=ATAN(QR/CR)
C
      WRITE(12,57)
      WRITE(12,55)
      WRITE(12,66)
      WRITE(12,56)
      WRITE(12,58)6,1
      WRITE(12,60)
      DO 27 I=1,L
      P=(I-1)*FRQ/2.
      Q=ATAN(QR(I)/CR(I))
```

```
      WRITE(12,59)P,Q
   27 CONTINUE
      WRITE(12,59)CC,CC
C
C     COMPUTE AUTOCORRELATION OF STRUCTURE RESPONSE Y
C     AND SPECTRAL DENSITY OF Y TO GET ESTIMATE OF
C     COHERENCE FUNCTION GAMMA=/GYX/**2/(GYY*GXX)
C
      CALL AUTOCO(Y,N,L,AR)
      CALL AUTOSP(L,H,AR,BR)
      WRITE(13,57)
      WRITE(13,55)
      WRITE(13,68)
      WRITE(13,56)
      WRITE(13,58)7,1
      WRITE(13,60)
      DO 28 I=1,L
      P=(I-1)*FRQ/2.
      Q=(CR(I)**2+QR(I)**2)/(G(I)*BR(I))
      WRITE(13,59)P,Q
   28 CONTINUE
      WRITE(13,59)CC,CC
   50 FORMAT(1X,'GROUND ACCELERATION')
   51 FORMAT(1X,'CORRELATION FUNCTIONS')
   52 FORMAT(1X,'SPECTRAL DENSITY FUNCTIONS')
   53 FORMAT(1X,'TIME (SEC)')
   54 FORMAT(1X,'LAG TIME (SEC)')
   55 FORMAT(1X,'FREQUENCY (CPS)')
   56 FORMAT(1X,'.',/1X,'.')
   57 FORMAT(1X,'.')
   58 FORMAT(1X,I3,/1X,I3)
   59 FORMAT(1X,3E16.6)
   60 FORMAT(1X,'1 LINE 1')
   61 FORMAT(1X,'AMPLITUDE OF FRF')
   62 FORMAT(1X,'OUTPUT DISPLACEMENT')
   63 FORMAT(1X,'RE')
   64 FORMAT(1X,'IM')
   65 FORMAT(1X,'5 LINE 1')
   66 FORMAT(1X,'PHASE ANGLE (RADIANS)')
   67 FORMAT(1X,I5)
   68 FORMAT(1X,'COHERENCE FUNCTION')
      STOP
      END
C
      SUBROUTINE AUTOCO(A,N,L,ANS)
C
C     TO FIND AUTOCOVARIANCES OF SERIES A FOR LAG 0 TO L-1
C     A   = ARRAY OF THE TIME SERIES
C     N   = NUMBER OF VALUES IN THE ARRAY A
C     L   = LAG TIME LENGTH
C     ANS = AUTOCOVARIANCE VALUES OF TIME SERIES A
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),ANS(1)
      FN=N
      AVE=0.
      DO 1 I=1,N
    1 AVE=AVE+A(I)
      AVE=AVE/FN
      AVE=AVE*AVE
      DO 3 J=1,L
      NJ=N-J+1
      FNJ=NJ
```

```
      SUM=0.
      DO 2 I=1,NJ
      IJ=I+J-1
    2 SUM=SUM+A(I)*A(IJ)
      SUM=SUM/FNJ
    3 ANS(J)=SUM-AVE
      RETURN
      END
C
C
      SUBROUTINE AUTOSP(M,H,R,G)
C
C     M = NUMBER OF POINTS
C     H = TIME INTERVAL
C     R = ARRAY OF AUTOCORELATION FUNCTION
C     G = ARRAY OF SPECTRAL DENSITY
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION R(1),G(1)
      COMMON/AAA/PI
      L=M-1
      ZN=1.
      DO 10 K=1,M
      P1=0.
      DO 11 I=2,M
   11 P1=P1+R(I)*COS(PI*(I-1.)*(K-1.)/L)
      G(K)=2.0*H*(R(1)+2.0*P1+ZN*R(M))
   10 ZN=-ZN
      P1=G(1)
      P2=G(2)
      P3=G(3)
      G(1)=0.5*(P1+P2)
      DO 12 K=2,L
      G(K)=0.25*(P1+2.0*P2+P3)
      P1=P2
      P2=P3
      P3=G(K+1)
   12 CONTINUE
      G(M)=0.5*(P2+P3)
      RETURN
      END
C
C
      SUBROUTINE CROSCO(A,B,N,L,AR,BR)
C
C     A  =  1-D ARRAY OF X TIME SERIE
C     B  =  1-D ARRAY OF Y TIME SERIE
C     N  =  NUMBER OF VALUES IN A AND B
C     L  =  LAG TIME LENGHT
C     AR =  (RXY+RYX)/2  - OUTPUT
C     BR =  (RXY-RYX)/2  - OUTPUT
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),B(1),AR(1),BR(1)
      FN=N
      AVEA=0.
      AVEB=0.
      DO 1 I=1,N
      AVEA=AVEA+A(I)
    1 AVEB=AVEB+B(I)
      AVEA=AVEA/FN
      AVEB=AVEB/FN
      AVAB=AVEA*AVEB
```

```
      DO 3 J=1,L
      NJ=N-J+1
      FNJ=NJ
      SR=0.
      SS=0.
      DO 2 I=1,NJ
      IJ=I+J-1
      SR=SR+A(I)*B(IJ)
    2 SS=SS+A(IJ)*B(I)
      BLA=SR/FNJ-AVAB
      BDA=SS/FNJ-AVAB
      AR(J)=0.5*(BLA+BDA)
    3 BR(J)=0.5*(BLA-BDA)
      RETURN
      END
C
C
      SUBROUTINE CROSSP(M,H,AR,BR,CR,QR)
C
C     M  =  NUMBER OF POINTS
C     H  =  TIME INTERVAL
C     AR =  ARRAY OF (RXY+RYX)/2
C     BR =  ARRAY OF (RXY-RYX)/2
C     CR =  COINCIDENT SDF
C     QR =  QUADRATURE SDF
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AR(1),BR(1),CR(1),QR(1)
      COMMON/AAA/PI
      L=M-1
      ZN=1.
      DO 10 K=1,M
      P1=0.
      Q1=0.
      DO 11 I=2,M
      T=PI*(I-1.)*(K-1.)/L
      P1=P1+AR(I)*COS(T)
   11 Q1=Q1+BR(I)*SIN(T)
      CR(K)=2.0*H*(AR(1)+2.0*P1+ZN*AR(M))
      QR(K)=4.0*H*Q1
   10 ZN=-ZN
      P1=CR(1)
      P2=CR(2)
      P3=CR(3)
      Q1=QR(1)
      Q2=QR(2)
      Q3=QR(3)
      CR(1)=0.5*(P1+P2)
      QR(1)=0.5*(Q1+Q2)
      DO 12 K=2,L
      CR(K)=0.25*(P1+2.0*P2+P3)
      QR(K)=0.25*(Q1+2.0*Q2+Q3)
      P1=P2
      P2=P3
      P3=CR(K+1)
      Q1=Q2
      Q2=Q3
      Q3=QR(K+1)
   12 CONTINUE
      CR(M)=0.5*(P2+P3)
      QR(M)=0.5*(Q2+Q3)
      RETURN
      END
```